CONTROL ENGINEERING 18ME71

Module-2
Modeling of Physical System

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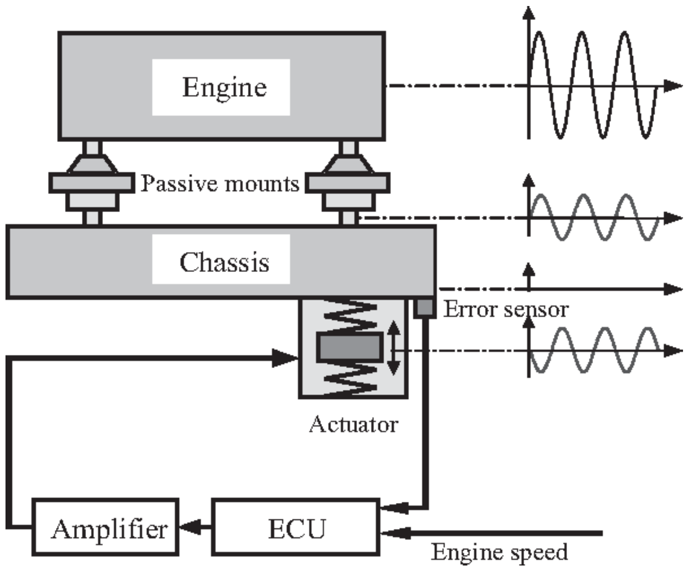
OBJECTIVES:

- To **develop** mathematical model for the mechanical, electrical and hydraulic systems.

Modeling of Control Systems

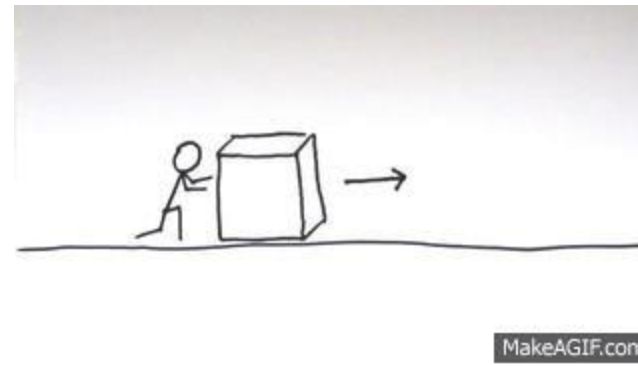
The process of obtaining the desired mathematical description of the system is called Mathematical Modeling.



Modeling of Mechanical Systems:

$$F_{\text{net external}} = ma$$

Net force on object = mass of object x acceleration

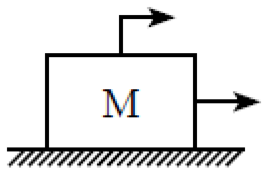


Mechanical systems can be of two types:

1) Translation Systems
2) Rotational Systems.

Translational system :

1. Mass :

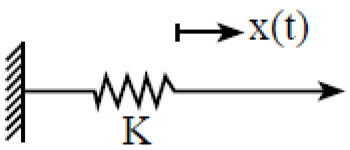


$\sum$F = mass × acceleration

$$F = M \times \frac{d^2x}{dt^2}$$

$$F = M \times x$$

2. Spring :

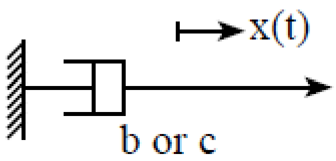


F = mass × acceleration

$$F - kx = 0$$

$$F = kx$$

3. Damper :



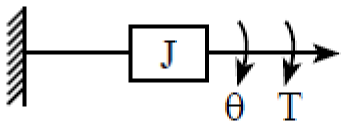
F = mass × acceleration

$$F - b\frac{dx}{dt} = 0$$

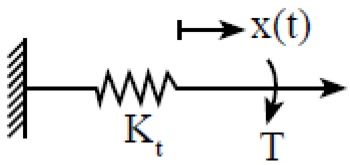
$$F = bx$$

Rotational system :

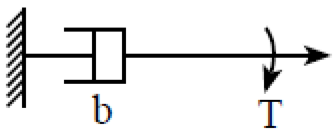
1. Rotating Mass :



2. Rotating Shaft (Spring):



3. Rotating damper:



$\sum$ T = Polar mass × Angular acceleration

$$T = J \times \frac{d^2\theta}{dy^2}$$

$$T = J \times \theta$$

$$\mathrm{T} = k_t \times \theta$$

$$\mathrm{T} = b \times \theta$$

Note : Laplace transformation**

$$(1)\, F(t) = F(s)$$

$$(2)\, x(t) = X(s)$$

$$(3)\, \frac{dx}{dt} @ \dot{x} = S\,X(s)$$

$$(4)\, \frac{d^2x}{dt^2} @ \ddot{x} = S^2\,X(s)$$

$$(5)\, \int x\,dt = \frac{1}{S}X(s)$$

$$(6)\, e^{-at} = \frac{1}{s+a}$$

Example 1 :

$$\sum F = \text{mass} \times \text{acceleration}$$

$$F - kx - b\frac{dx}{dt} = M \times \frac{d^2x}{dt^2}$$

$$F - kx - bx = Mx$$

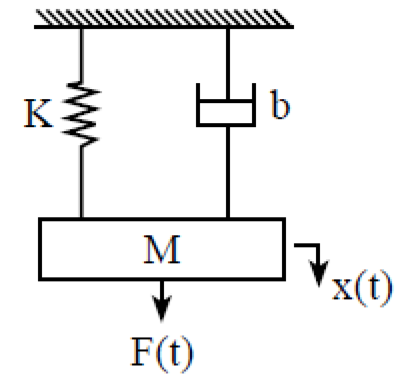
$$F = Mx + bx + kx$$

Taking Laplace transformation

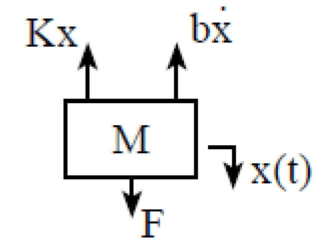
$$F(s) = MS^2X(s) + bSX(s) + k\,X(s)$$

$$F(s) = X(s)^*MS^2 + bS + k\}$$

$$Transfer\ Function = \frac{X(s)}{F(s)} = \frac{1}{{}^*MS^2 + bS + k\}}$$

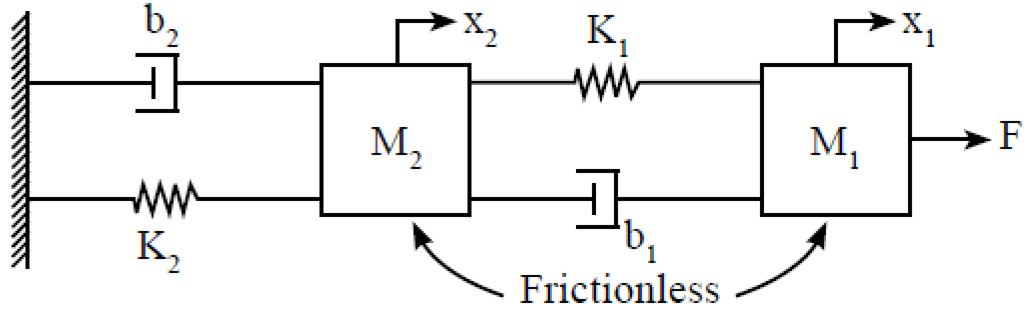


Free body diagram



$$Transfer\ Function = \frac{Output}{Input}$$

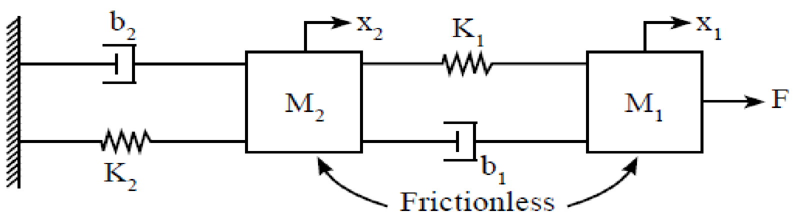
Problem 1: Obtain differential equation and transfer function for the system.



Note :

1. While analysing mass M_1, assume $x_1 > x_2$ and we have to take relative velocity and relative displacement.
2. While analysing mass M_2, assume $x_2 > x_1$ and we have to consider relative velocity and relative displacement.

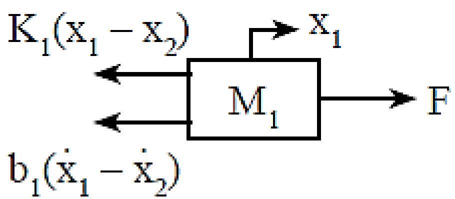
Analysing mass M_1 $(x_1 > x_2)$



$\sum F$ = mass × acceleration

$$F - k_1(x_1 - x_2) - b_1(x_1 - x_2) = M_1 x_1$$

Free body diagram :



$$F - k_1 x_1 + k_1 x_2 - b_1 x_1 + b_1 x_2 = M_1 x_1$$

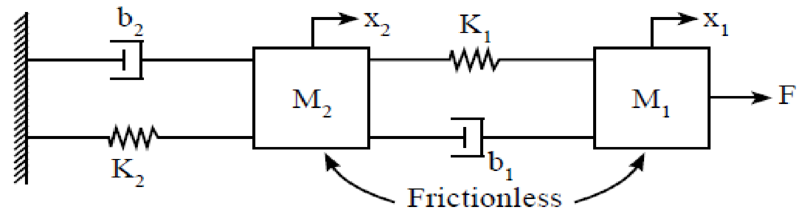
$$F + k_1 x_2 + b_1 x_2 = M_1 x_1 + b_1 x_1 + k_1 x_1 \qquad \text{---(A)}$$

Taking Laplace Transform,

$$F(s) + k_1 X_2(s) + b_1 s X_2(s) = M_1 s^2 X_1(s) + b_1 s X_1(S) + k_1 X_1(s)$$

$$\boxed{F(s) + X_2(s),b_1 s + k_1- = X_1(s),M_1 s^2 + b_1 s + k_1-} \text{---(Eq 1)}$$

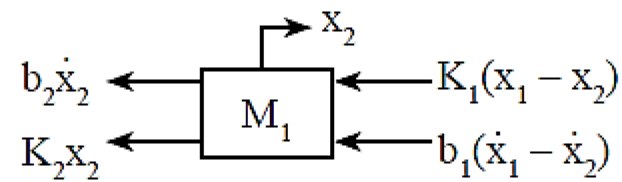
Analysing mass M_2 $(x_2 > x_1)$



$\sum F = \text{mass} \times \text{acceleration}$

$$-b_2 x_2 - k_2 x_2 - k_1(x_2 - x_1) - b_1(x_2 - x_1) = M_2 x_2$$

Free body diagram :



$$b_1 x_1 + k_1 x_1 = M_2 x_2 + b_2 x_2 + b_1 x_2 + k_2 x_2 + k_1 x_2$$ ---(B)

Taking Laplace Transform,

$$b_1 s X_1(s) + k_1 X_1(s) = M_2 s^2 X_2(s) + b_2 s X_2(s) + b_1 s X_2(s) + k_2 X_2(s) + k_1 X_2(s)$$

$$\boxed{X_1(s)[b_1 s + k_1] = X_2(s)[M_2 s^2 + b_2 s + b_1 s + k_2 + k_1]}$$ ---(Eq 2)

$$X_1(s) = \frac{F(s) + X_2(s)[b_1 s + k_1]}{,M_1 s^2 + b_1 s + k_1\text{-}} \text{ ---(Eq 1)} \qquad X_1(s) = X_2(s)\frac{,M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-}}{,b_1 s + k_1\text{-}} \text{---(Eq 2)}$$

$$\frac{F(s) + X_2(s)[b_1 s + k_1]}{,M_1 s^2 + b_1 s + k_1\text{-}} = X_2(s)\frac{,M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-}}{,b_1 s + k_1\text{-}}$$

$$F(s) = X_2(s)\frac{,M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-} \times ,M_1 s^2 + b_1 s + k_1\text{-}}{,b_1 s + k_1\text{-}} - X_2(s)[b_1 s + k_1]$$

$$F(s) = \frac{X_2(s) *,M_1 s^2 + b_1 s + k_1\text{-},M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-} - [b_1 s + k_1]^2 +}{,b_1 s + k_1\text{-}}$$

$$\textit{Transfer Function} = \frac{\textit{Output}}{\textit{Input}} = \frac{X_2(s)}{F(s)} = \frac{,b_1 s + k_1\text{-}}{*,M_1 s^2 + b_1 s + k_1\text{-},M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-} - [b_1 s + k_1]^2 +}$$

$$\frac{X_1(s)[M_1s^2 + b_1s + k_1] - F(s)}{(b_1s + k_1)} = X_2(s) \text{ ---(Eq 1)} \qquad X_2(s) = \frac{(b_1s + k_1)X_1(s)}{(M_2s^2 + b_2s + b_1s + k_2 + k_1)} \text{---(Eq 2)}$$

$$\frac{X_1(s)[M_1s^2 + b_1s + k_1] - F(s)}{(b_1s + k_1)} = \frac{(b_1s + k_1)X_1(s)}{(M_2s^2 + b_2s + b_1s + k_2 + k_1)}$$

$$X_1(s)[M_1s^2 + b_1s + k_1] - F(s) = \frac{(b_1s + k_1)^2X_1(s)}{(M_2s^2 + b_2s + b_1s + k_2 + k_1)}$$

$$F(s) = X_1(s)[M_1s^2 + b_1s + k_1] - \frac{(b_1s + k_1)^2X_1(s)}{(M_2s^2 + b_2s + b_1s + k_2 + k_1)}$$

$$F(s) = \frac{\{X_1(s)[M_1s^2 + b_1s + k_1](M_2s^2 + b_2s + b_1s + k_2 + k_1)\} - (b_1s + k_1)^2X_1(s)}{(M_2s^2 + b_2s + b_1s + k_2 + k_1)}$$

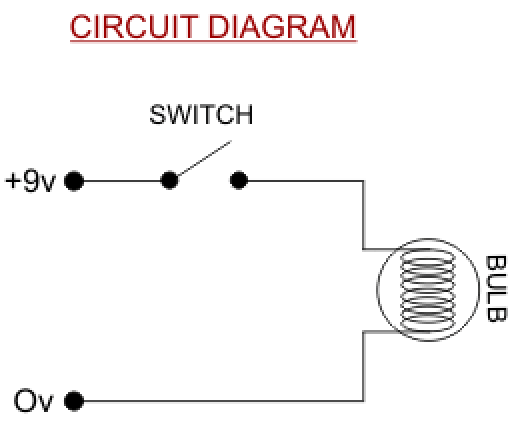
$$F(s) = \frac{\{X_1(s)[M_1 s^2 + b_1 s + k_1],M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-}\} - ,b_1 s + k_1\text{-}^2 X_1(s)}{,M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-}}$$

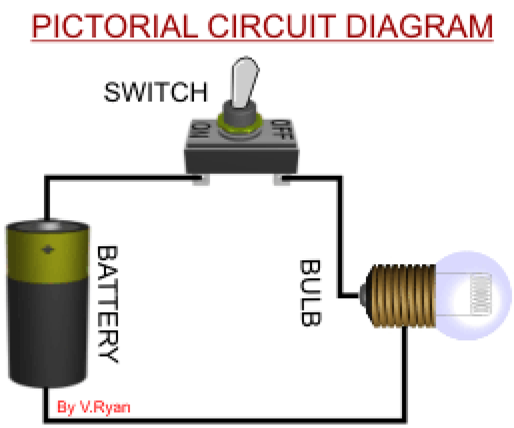
$$F(s) = \frac{X_1(s)\{[M_1 s^2 + b_1 s + k_1][M_2 s^2 + b_2 s + b_1 s + k_2 + k_1] - ,b_1 s + k_1\text{-}^2\}}{,M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-}}$$

$$\textit{Transfer Function} = \frac{X_1(s)}{F(s)} = \frac{[M_2 s^2 + b_2 s + b_1 s + k_2 + k_1]}{\{[M_1 s^2 + b_1 s + k_1][M_2 s^2 + b_2 s + b_1 s + k_2 + k_1] - ,b_1 s + k_1\text{-}^2\}}$$

$$\textit{Transfer Function} = \frac{Output}{Input} = \frac{X_2(s)}{F(s)} = \frac{,b_1 s + k_1\text{-}}{*,M_1 s^2 + b_1 s + k_1\text{-},M_2 s^2 + b_2 s + b_1 s + k_2 + k_1\text{-} - [b_1 s + k_1]^2 +}$$

Modelling
of
Electrical System



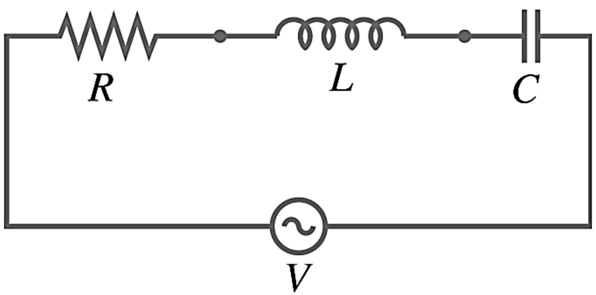


Example 1: Obtain differential equation for R-L-C circuit

According Kirchhoff's voltage law

$$V = V_R + V_L + V_C$$

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{---Eq (1)}$$



Applying Laplace Transform

$$V(s) = RI(s) + LsI(s) + \frac{1}{Cs}I(s)$$

$$V(s) = \frac{(Ls^2C + RCs + 1)}{Cs}I(s)$$

$$\boxed{\textit{Transfer Function} = \frac{I(s)}{V(s)} = \frac{Cs}{(Ls^2C + RCs + 1)}}$$

Note : Current **'i'** is given as rate of change of charge (q) with respect to time

$$i = \frac{dq}{dt}$$

$$\frac{di}{dt} = \frac{d^2q}{dt^2}$$

$$\int i\,dt = q$$

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}q$$

Armature controlled DC motor :

Let

V_a = Armature voltage

i_a = Armature current

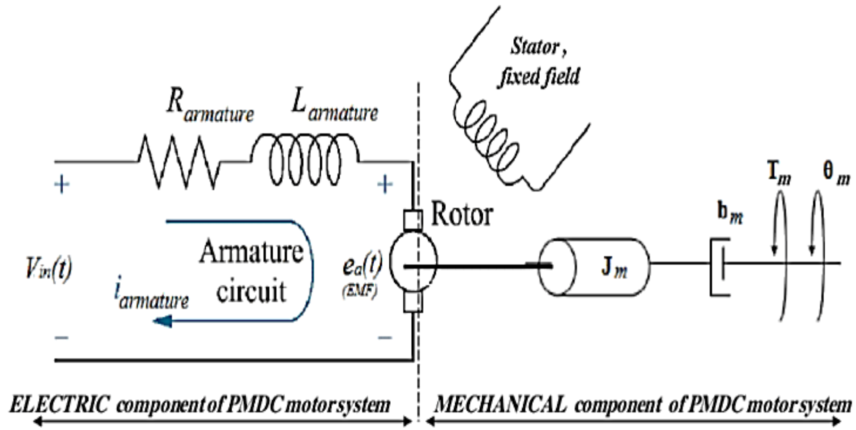
R_a = Armature resistance

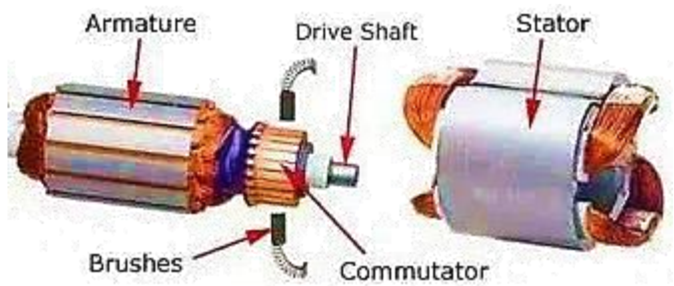
L_a = Armature inductance

i_f = Field current

J = Inertia mass

b = Damping co-efficient





Area of flux is proportional to field current

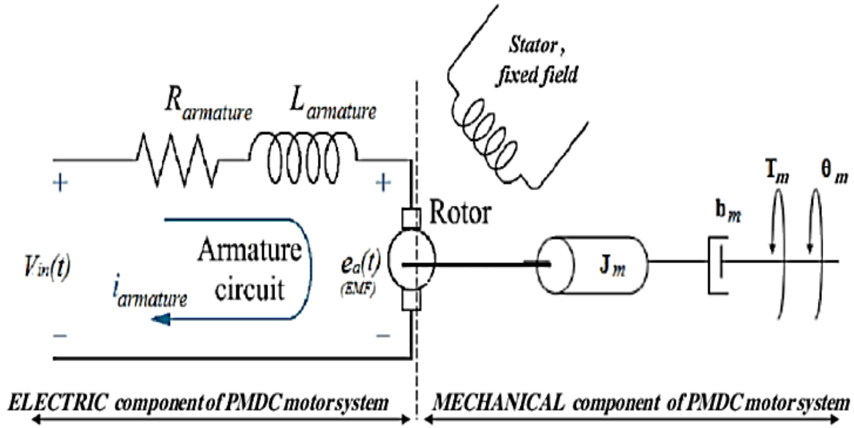
$$\emptyset \; \alpha \; i_f$$

$$\emptyset = k_f \times i_f \quad \text{--Eq (1)}$$

k_f = Field constant

Armature controlled DC motor :

Also, the torque developed by the motor is proportional to product of armature current and area of flux.



$$T \,\alpha\, i_a\, \emptyset$$

$$T = k_1\, i_a k_f \times i_f$$

$$T = k_t\, i_a \quad \text{--Eq (2)}$$

$$\boldsymbol{k_t = k_1 k_f k_f}$$

If, back e.m.f. is proportional to speed of the motor then,

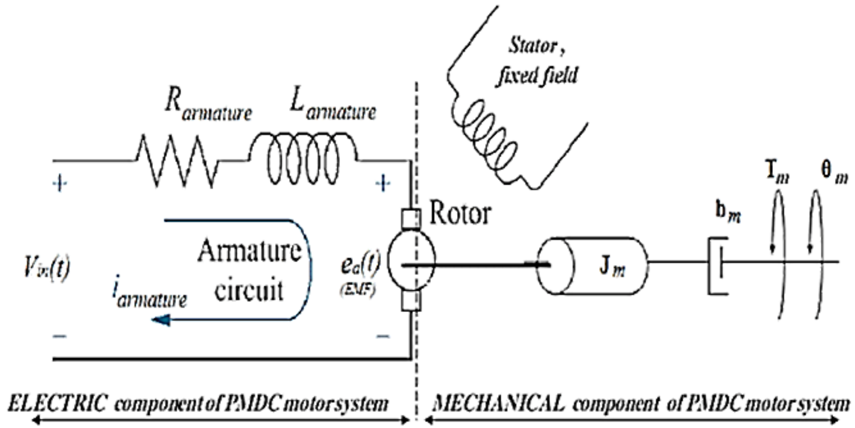
$$e_b = k_b \frac{d\theta}{dt} \quad \text{--Eq (3)}$$

Armature controlled DC motor :

Speed of the motor is controlled by armature voltage, by applying Kirchhoff 's voltage law to armature circuit

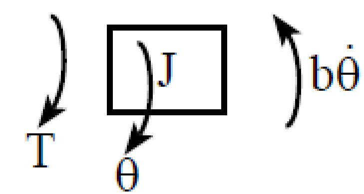
$$V_a = V_R + V_L + e_b$$

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + k_b \frac{d\theta}{dt} \qquad \text{--Eq (4)}$$



The armature current produces torque which is applied to mechanical load

By Newton second law of motion



$$\sum T = J\,\theta$$

$$T - b\theta = J\,\theta$$

$$T = J\theta + b\theta \qquad \text{--Eq (5)}$$

Armature controlled DC motor :

Equating Eq (2) and Eq (5)

$$k_t\, i_a = J\theta + b\theta \quad \Rightarrow \quad i_a = \frac{,J\theta + b\theta-}{k_t}$$

Substituting value of "i_a" in Eq (4)

$$V_a = R_a \frac{,J\theta + b\theta-}{k_t} + L_a \frac{d}{dt}\left\{\frac{,J\theta + b\theta-}{k_t}\right\} + k_b \frac{d\theta}{dt}$$

$$V_a = \frac{R_a}{k_t},J\theta + b\theta- + \frac{L_a}{k_t},J\theta + b\theta- + k_b\theta$$

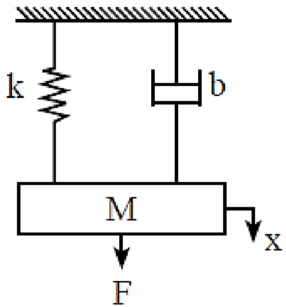
Applying Laplace Transform

$$V_a(s) = \frac{R_a}{k_t},Js^2\,\theta(s) + bs\theta(s)- + \frac{L_a}{k_t},Js^3\theta(s) + bs^2\theta(s)- + k_b s\theta(S)$$

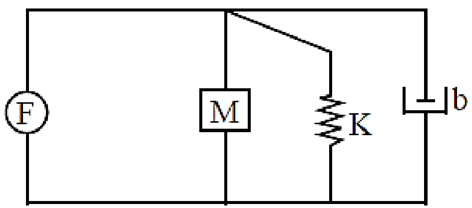
$$\boxed{\textit{Transfer Function} = \frac{\theta(s)}{V_a(s)} = \frac{k_t}{L_a[Js^3 + bs^2] + R_a,Js^2 + bs- + k_b k_t s}}$$

Analogy between Mechanical and Electrical network

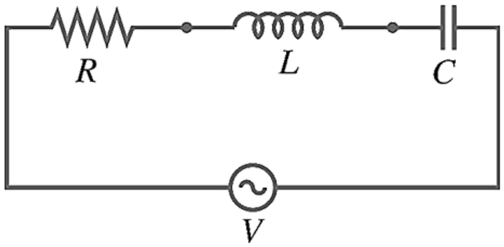
1. Force voltage analogy [FV Analogy] :



Mechanical Network



Electrical Network



By Newton's second law of motion

$$F = Mx + bx + kx$$ ---Eq (1)

By Kirchhoff voltage law

$$V = Lq + Rq + \frac{q}{C}$$ ---Eq (2)

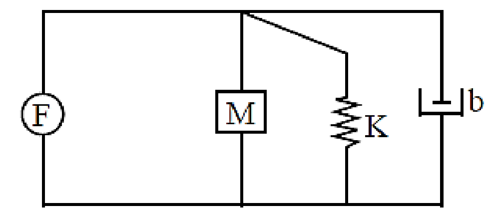
$$F = Mx + bx + kx \quad \text{---Eq (1)}$$

$$V = Lq + Rq + \frac{q}{C} \quad \text{---Eq (2)}$$

Mechanical system	Electrical system
Force (F)	Voltage (V)
Mass (M)	Inductance (L)
Damper (b)	Resistance (R)
Spring stiffness (k)	Reciprocal of capacitance (1/C)
Displacement (x)	Flow of charge (q)

2. Force current analogy [F - I Analogy]

Mechanical Network



By Kirchhoff Current law

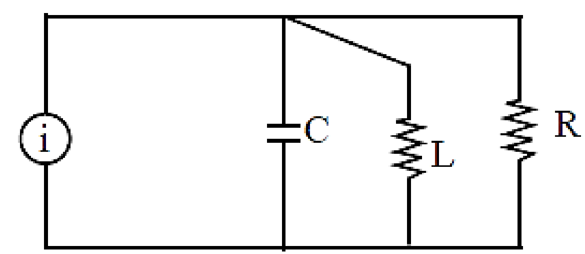
$$i = i_c + i_L + i_R$$

$$i = C\frac{dv}{dt} + \frac{1}{L}\int v\,dt + \frac{v}{R}$$

$$v = \frac{d\emptyset}{dt} \Rightarrow \frac{dv}{dt} = \frac{d^2\emptyset}{dt^2} \Rightarrow \int v\,dt = \emptyset$$

$$i = C\frac{d^2\emptyset}{dt^2} + \frac{1}{R}\frac{d\emptyset}{dt} + \frac{1}{L}\emptyset$$

$$i = C\emptyset + \frac{1}{R}\emptyset + \frac{1}{L}\emptyset$$ ---Eq (3)



$$F = Mx + bx + kx$$ ---Eq (1)

$$i = C\emptyset + \frac{1}{R}\emptyset + \frac{1}{L}\emptyset$$ ---Eq (3)

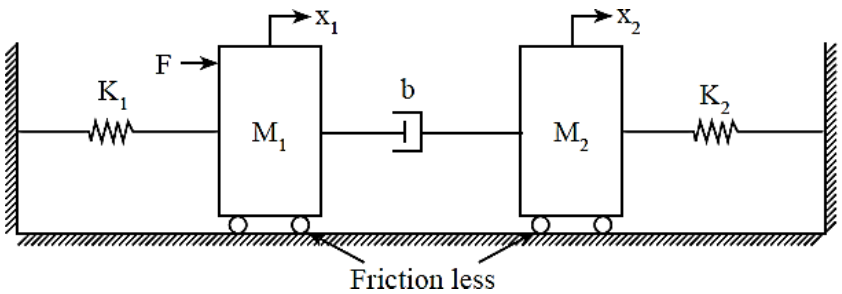
Mechanical system	Electrical system
Force (F)	Current (i)
Mass (M)	Capacitance (C)
Damper (b)	Reciprocal of resistance (1/R)
Spring stiffness (k)	Reciprocal of inductance(1/L)
Displacement (x)	Flux quantity(Ø)

Analogy :

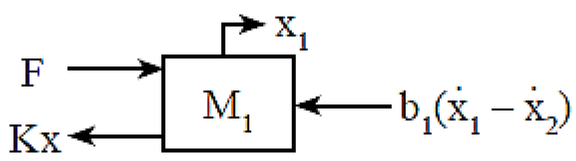
System Type	Flow Variable	Effort Variable	Compliance	Inductance	Resistance
Mechanical	X =*Displacement*	F = Force	Spring (K)	Mass (M)	Damper (C)
Electrical	I = Current	V = Voltage	Capacitance (C)	Inductance (L)	Resistance (R)
Thermal	Q_h = Heat Flow Rate	ΔT = Change In Temperature	Object (C)	Inductance (L)	Conduction & Convection (R)
Fluid	Q_m = Mass Flow Rate, q_v = Volume Flow Rate	P = Pressure, H = Height	Tank (C)	Mass (M)	Valve (R)

Flow variable: Moves through the system
Effort variable: Puts the system into action
Compliance: Stores energy as potential
Inductance: Stores energy as kinetic
Resistance: Dissipates or uses energy

Example 1: Draw mechanical network for system shown in figure. Write force voltage (FV) and force current (FI) analogy.



Considering mass $M_1(x_1 > x_2)$:

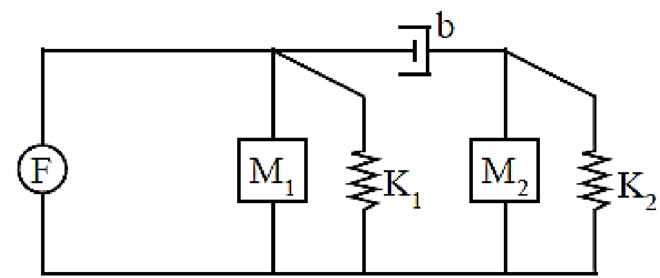


$$\sum F = Mx$$

$$F - k_1 x_1 - b\,(x_1 - x_2) = M_1 x_1$$

$$F = M_1 x_1 + b\,(x_1 - x_2) + k_1 x_1 \quad \text{---Eq (1)}$$

Equivalent mechanical network



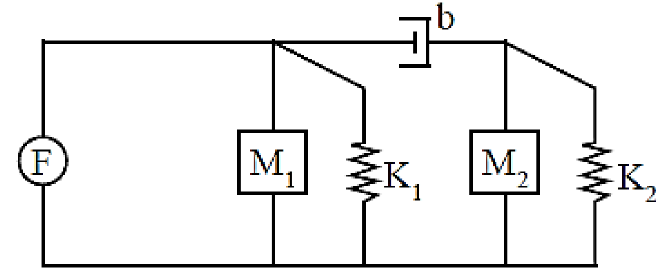
Considering mass $M_1(x_1 > x_2)$:

$$\sum F = Mx$$

$$-b\,(x_2 - x_1) - k_2 x_2 = M_2 x_2$$

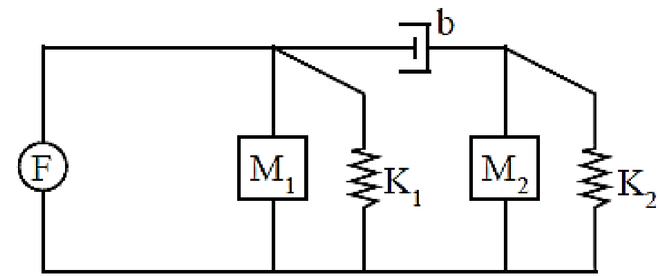
$$0 = M_2 x_2 + b\,(x_2 - x_1) + k_2 x_2 \quad \text{---Eq (2)}$$

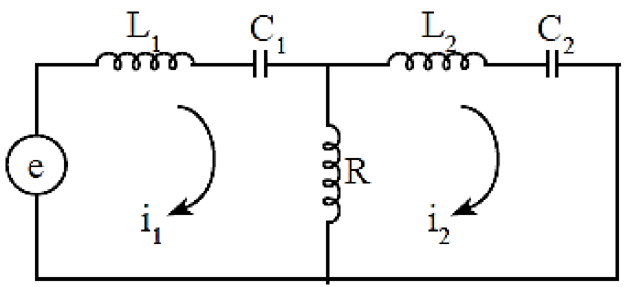
Equivalent mechanical network



FV Analogy :

Equivalent mechanical network





Loop (1) : ($i_1 > i_2$)

By Kirchhoff's voltage law

$$v = v_{L1} + v_{C1} + v_R$$

$$v = L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1 . dt + R(i_1 - i_2) \quad \text{---Eq (3)}$$

Loop (2) : ($i_2 > i_1$)

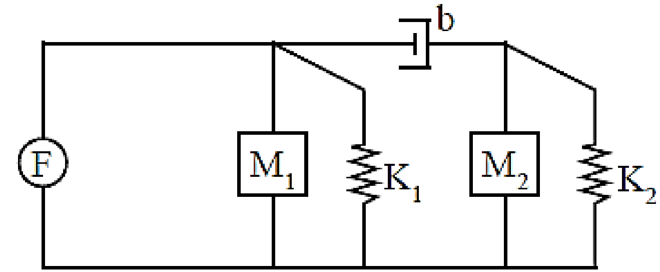
By Kirchhoff's voltage law

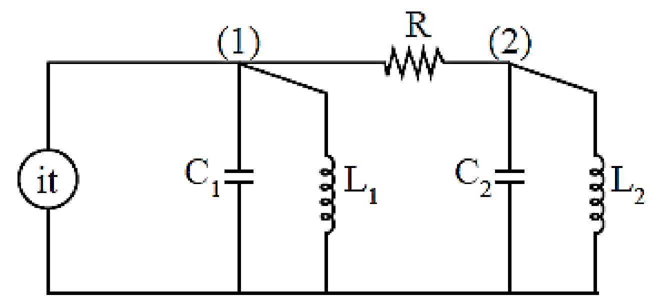
$$v = v_R + v_{L2} + v_{C2}$$

$$0 = R(i_2 - i_1) + L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2 . dt \quad \text{---Eq (4)}$$

FI Analogy :

Equivalent mechanical network





Loop (1) : ($V_1 > V_2$)

By Kirchhoff's voltage law

$$i = i_{C1} + i_{L1} + i_R$$

$$i = C_1 \frac{dv_1}{dt} + \frac{1}{L_1}\int v_1.dt + \frac{1}{R}(v_1 - v_2) \text{---Eq (5)}$$

Loop (2) : ($V_2 > V_1$)

By Kirchhoff's voltage law

$$i = i_R + i_{C2} + i_{L2}$$

$$0 = \frac{1}{R}(v_2 - v_1) + C_2 \frac{dv_2}{dt} + \frac{1}{L_2}\int v_2.dt \text{---Eq (6)}$$

Analogy :

$$F = M_1 x_1 + b\,(x_1 - x_2) + k_1 x_1 \quad \text{---Eq (1)}$$
$$0 = M_2 x_2 + b\,(x_2 - x_1) + k_2 x_2 \quad \text{---Eq (2)}$$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Mechanical network}$$

$$v = L_1 \frac{di_1}{dt} + \frac{1}{C_1} \int i_1 . dt + R(i_1 - i_2) \quad \text{---Eq (3)}$$
$$0 = R(i_2 - i_1) + L_2 \frac{di_2}{dt} + \frac{1}{C_2} \int i_2 . dt \quad \text{---Eq (4)}$$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Voltage network}$$

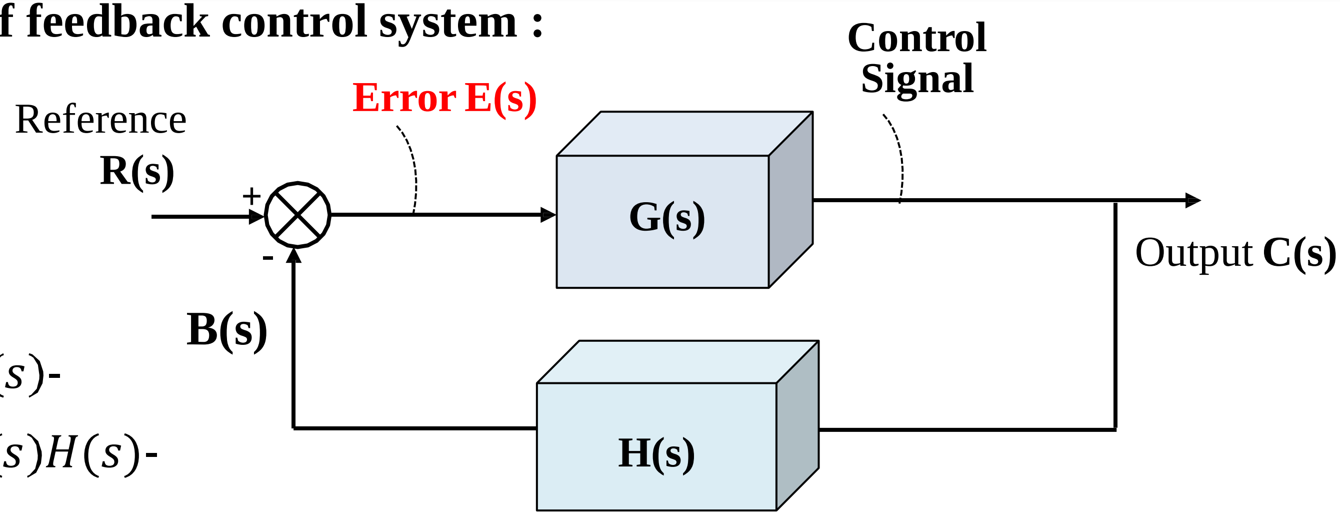
$$i = C_1 \frac{dv_1}{dt} + \frac{1}{L_1} \int v_1 . dt + \frac{1}{R}(v_1 - v_2) \text{---Eq (5)}$$
$$0 = \frac{1}{R}(v_2 - v_1) + C_2 \frac{dv_2}{dt} + \frac{1}{L_2} \int v_2 . dt \text{---Eq (6)}$$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Current network}$$

Block Diagram Algebra

Representation of ***mathematical models*** by applying appropriate laws, and then it is place inside a ***block***, and these blocks are ***connected by arrows*** to show the direction or flow of signals, the diagram thus obtained is called as "***Block diagram***".

General representation of feedback control system :



$$C(s) = G(s)E(s)$$

$$C(s) = G(s)[R(s) \pm B(s)]$$

$$C(s) = G(s)[R(s) \pm C(s)H(s)]$$

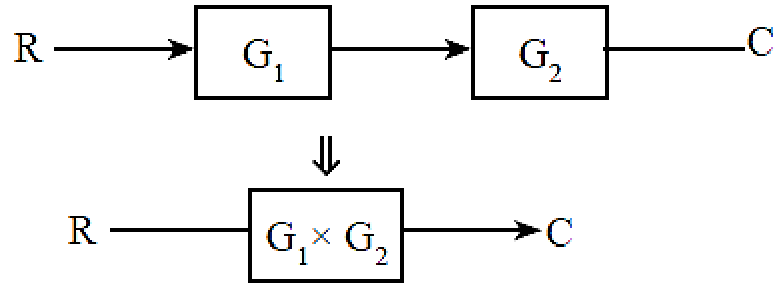
$$C(s) = G(s)R(s) \pm C(s)H(s)G(s)$$

$$G(s)R(s) = C(s) \pm C(s)G(s)H(s)$$

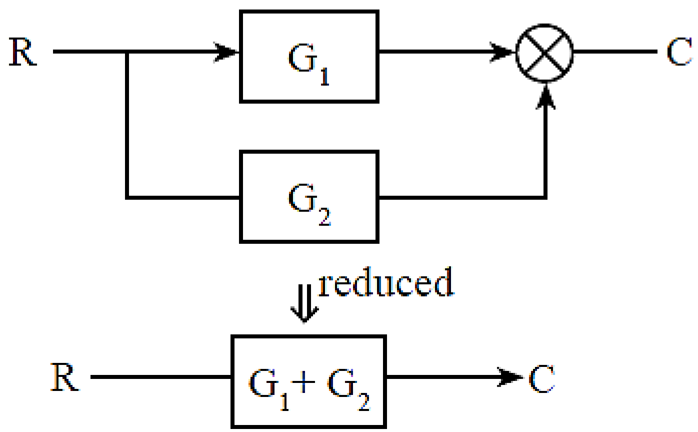
$$\frac{C(s)}{R(s)} = \frac{G(s)}{[1 + G(s)H(s)]}$$

Rules of block diagram reduction technique :

1. Block in series :

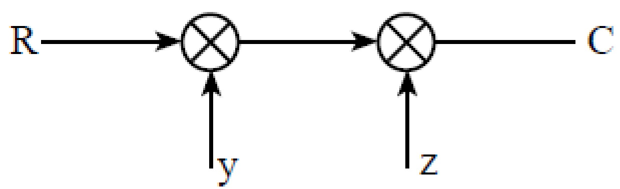


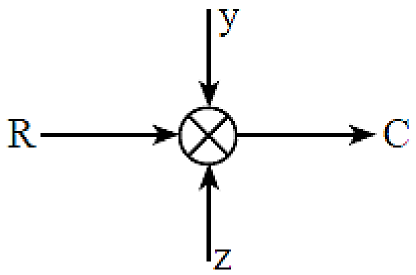
2. Blocks in parallel :



Rules of block diagram reduction technique :

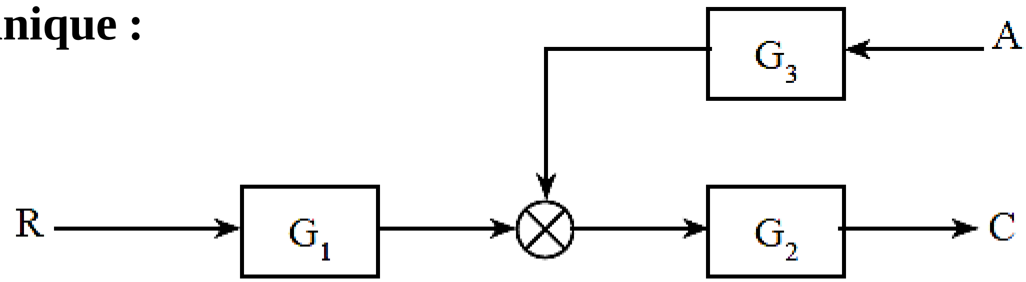
3. Merging of two summing point :





Rules of block diagram reduction technique :

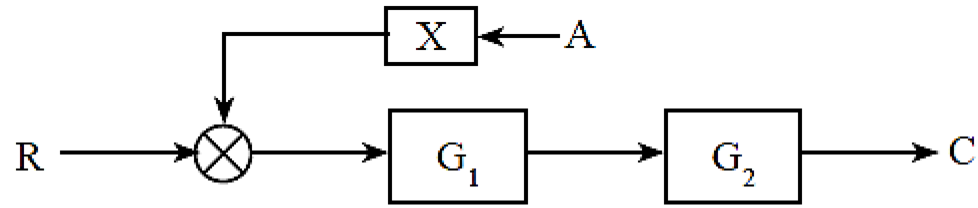
4. Shifting of summing point :



$$C = (RG_1 + AG_3)G_2$$

$$C = RG_1G_2 + AG_2G_3 \text{----Eq (1)}$$

4. a. Shifting of summing point against signal :



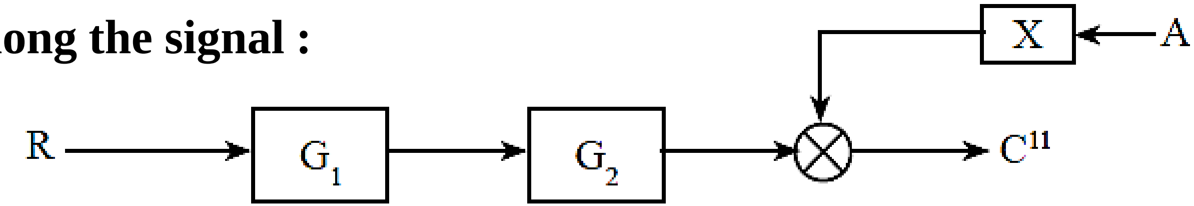
$$C^1 = RG_1G_2 + AXG_1G_2 \quad \text{----Eq (2)}$$

$$RG_1G_2 + AXG_1G_2 = RG_1G_2 + AG_2G_3$$

$$X = \frac{G_3}{G_1}$$

Rules of block diagram reduction technique :

4. b. Shifting of summing point along the signal :



$$C^{11} = RG_1G_2 + AX \qquad \text{----Eq (3)}$$

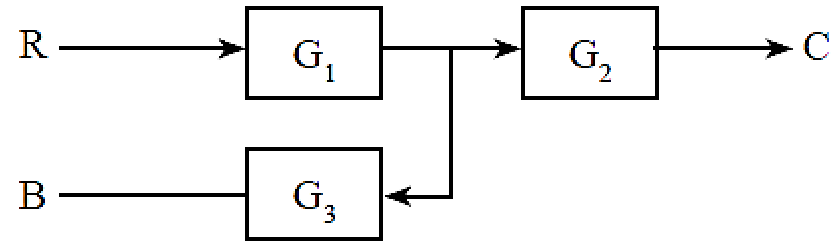
$$RG_1G_2 + AX = RG_1G_2 + AG_2G_3$$

$$X = G_2G_3$$

- ❖ If the summering point is shaft against the signal then the value of block is divided by the shifted block
- ❖ If the summary point is shifted along the signal then the value of block is multiplied by the shifted block.

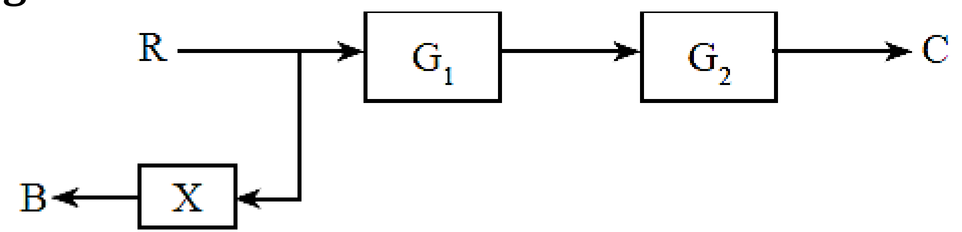
Rules of block diagram reduction technique :

5. Moving of take off point :



$B = RG_1G_3$ ----Eq (4)

5. a. Moving take off point against the signal :



$B^1 = RX$ ----Eq (5)

$RX = RG_1G_3$

$X = G_1G_3$

If take off point is moved against the signal then the value of block is taken as a product of the block above which is shifted.

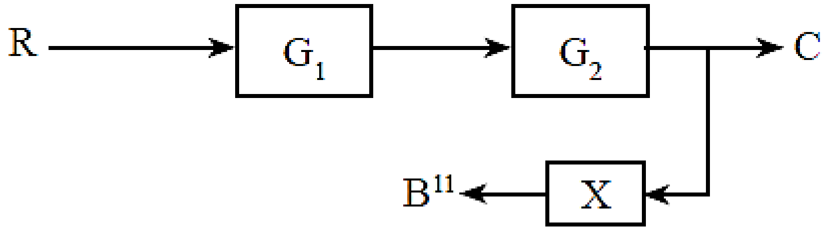
Rules of block diagram reduction technique :

5. b. Moving takeoff point along the signal.

$$B^{11} = RXG_1G_2 \text{ ----Eq (6)}$$

$$RG_1G_3 = RXG_1G_2$$

$$X = \frac{G_3}{G_2}$$



If take off point is moved along the signal then the value of block is taken as a product of reciprocal of the block above which is shifted.

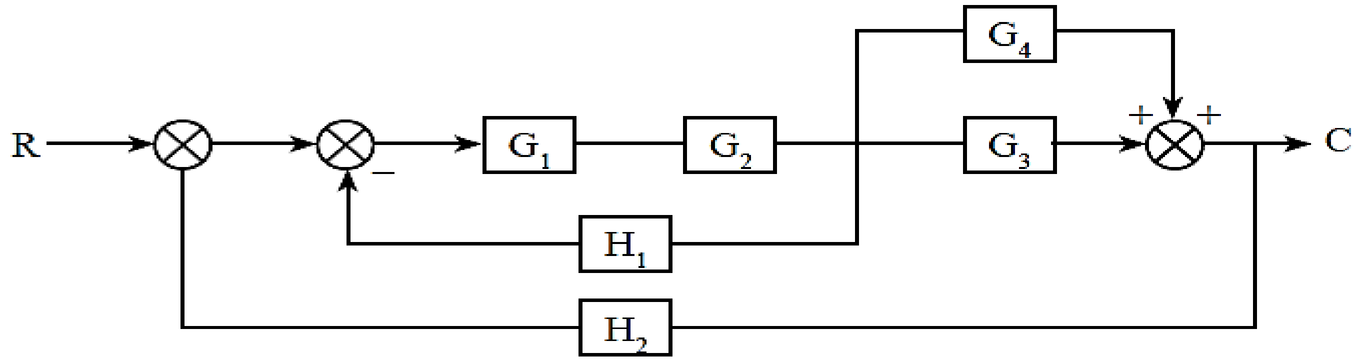
Steps for block diagram reduction :

1. Combine block in series and block in parallel.
2. Eliminate minor loop using the formula

$$\frac{C(s)}{R(s)} = \frac{1}{\{1 + G(s)H(s)\}}$$

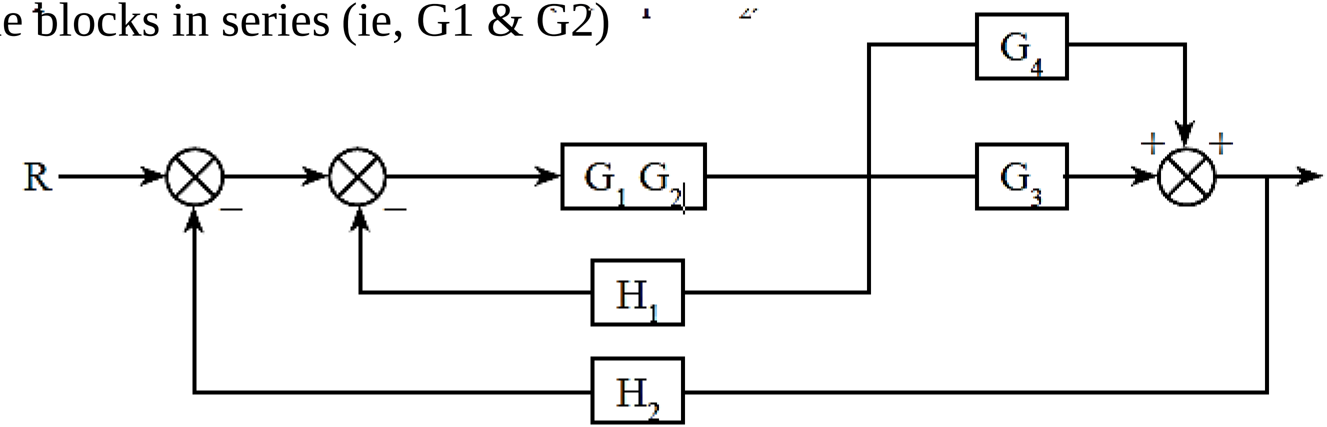
3. As far as possible try to shift take off point along the signal and summing point against the signal.
4. The above steps are carried out until we obtain control ratio.
5. Do not bring take off point and summing point side by side.
6. Also do not move take off point around summing point and vice versa.

Example 1: Reduce the block diagram using block diagram reduction technique and obtain control ratio.

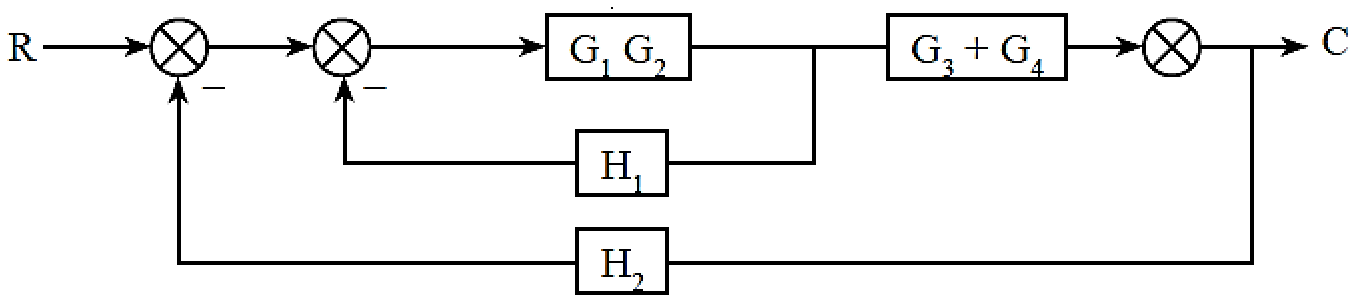


Solution.

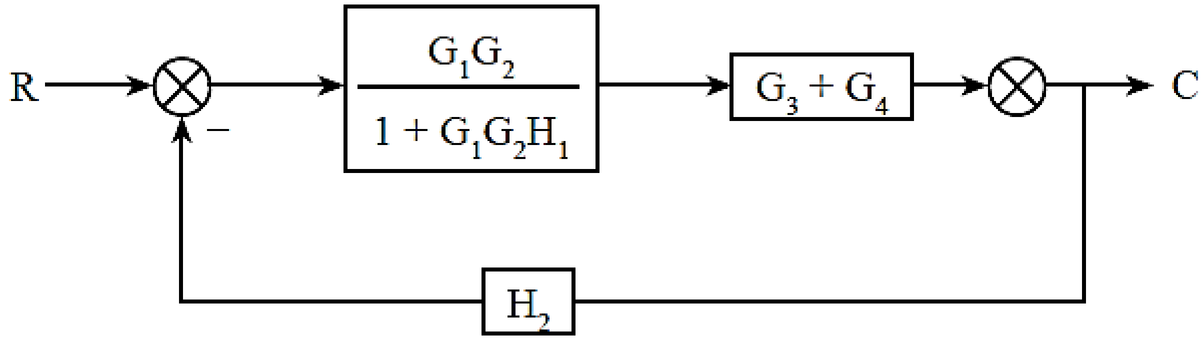
Step 1: Combine blocks in series (ie, G1 & G2)



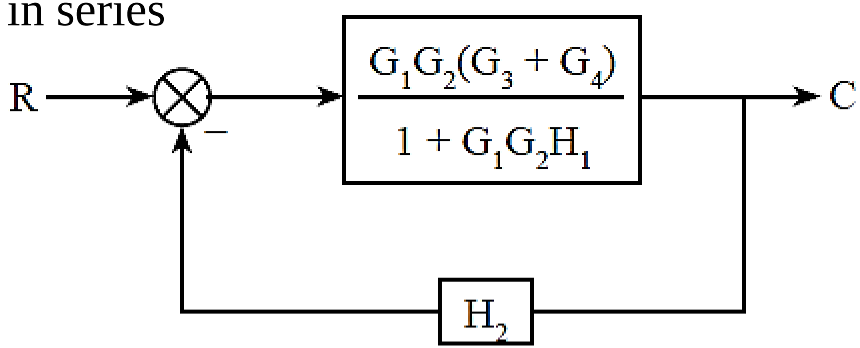
Step 2 Combine the blocks in parallel



Step 3 Eliminate the minor loop



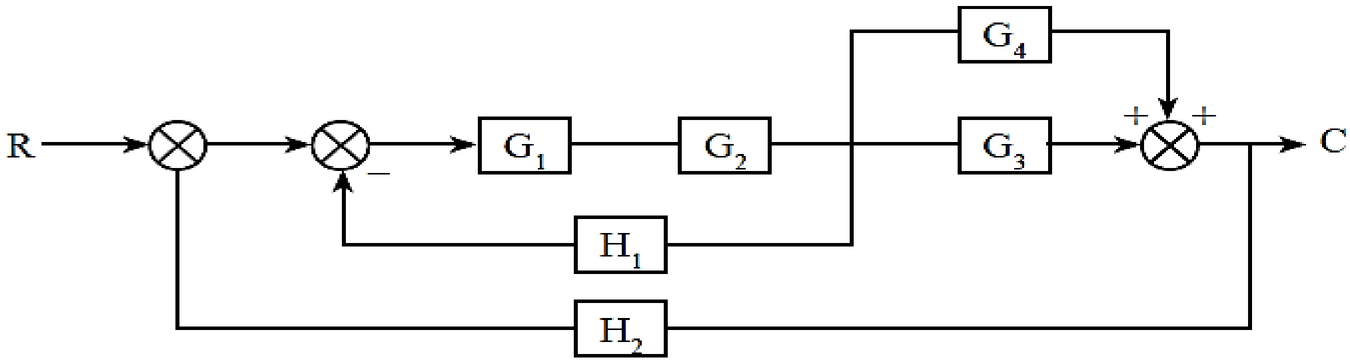
Step 4 Combine blocks in series



Step 5 Eliminating loop

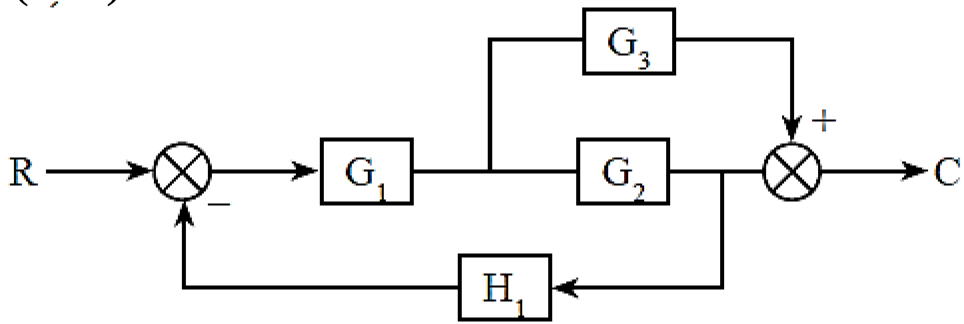
$$\frac{C}{R}=\frac{\dfrac{G_1G_2\left(G_3+G_4\right)}{1+G_1G_2H_1}}{1+\dfrac{G_1G_2\left(G_3+G_4\right)}{1+G_1G_2H_1}\times H_2}$$

$$\boxed{\frac{C}{R}=\frac{G_1G_2G_3+G_1G_2G_4}{1+G_1G_2H_1+G_1G_2G_3H_2+G_1G_2G_4H_1}}$$

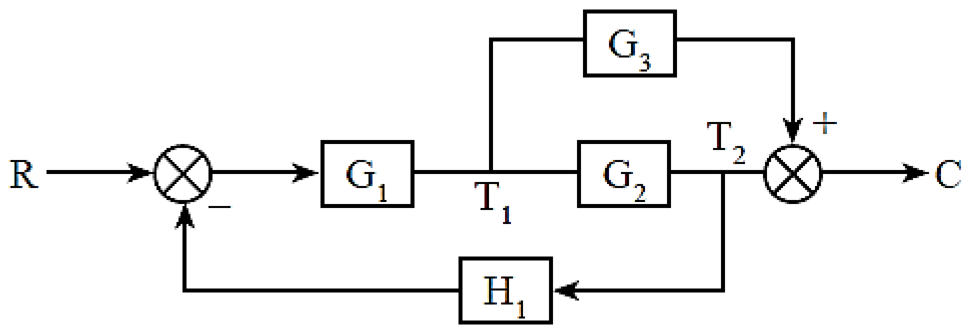


$$R \longrightarrow \boxed{\dfrac{G_1G_2G_3 + G_1G_2G_4}{1 + G_1G_2H_1 + G_1G_2G_3H_2 + G_1G_2G_4H_1}} \longrightarrow C$$

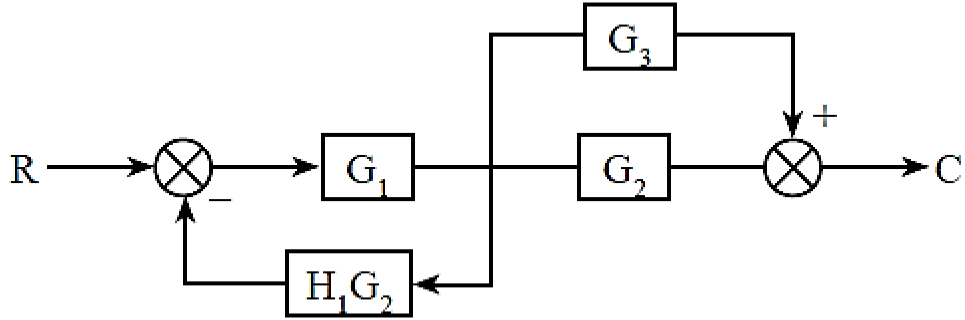
Example 2 : Reduce the block diagram using block diagram reduction technique and obtain control ratio(C/R).



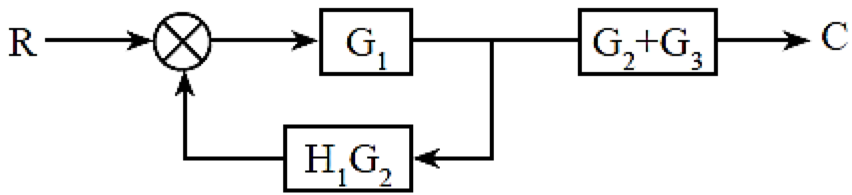
In this problem take off point in between the blocks hence we need to move take off point.



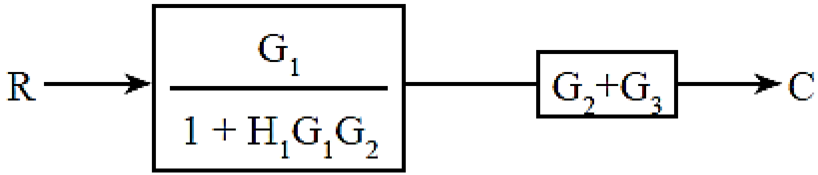
Step 1: Shifting of takeoff point (T_2) against the signal around block



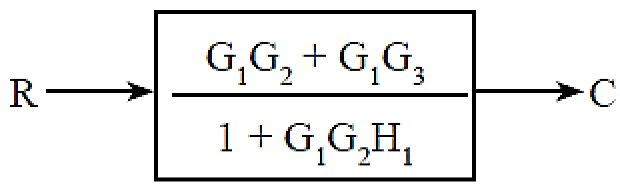
Step 2: Combine blocks in parallel



Step 3: Eliminating loop



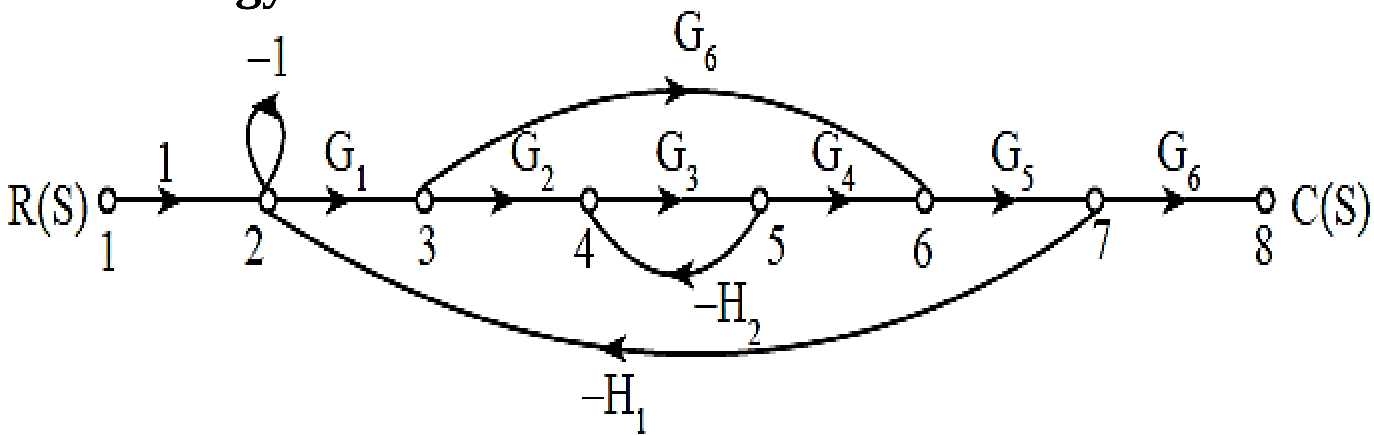
Step 4: Combine block in series



Signal Flow Graph

- Alternative method to block diagram representation, developed by ***Samuel Jefferson Mason.***
- Advantage: the availability of a flow graph gain formula, also called Mason's gain formula.
- A signal-flow graph consists of a network in which nodes are connected by directed branches.
- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Important terminology



1. Branches
2. Input node or source node
3. Output or sink node
4. Mixed node
5. Self loop
6. Forward path
7. Forward path gain
8. Loop
9. Loop gain
10. Non - touching loops

Mason's Gain Formula

The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{k=1}^{n} P_k\Delta_k \qquad \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + \_ + P_n\Delta_n}{\Delta}$$

Where

k = number of forward paths.
P_k = the k th forward-path gain.
Δ = Determinant of the system
Δ_k = Determinant of the k th forward path

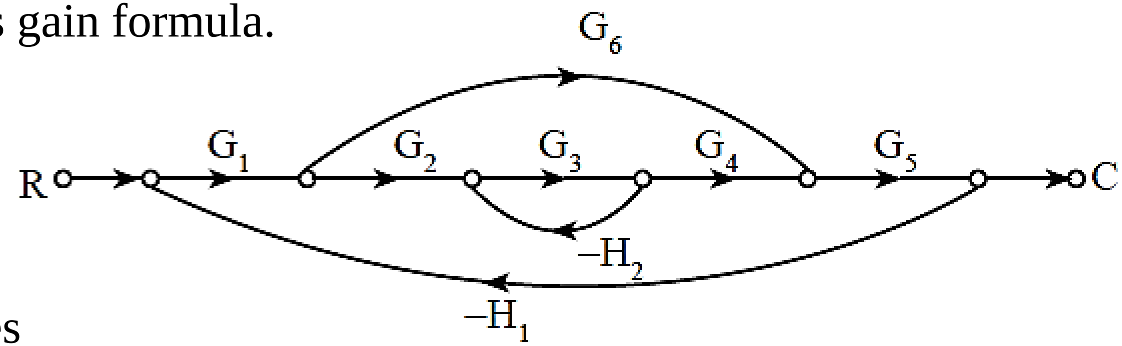
$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{k=1}^{n} P_k\Delta_k$$

Δ is called the signal flow graph determinant or characteristic function. Since Δ=0 is the system characteristic equation.

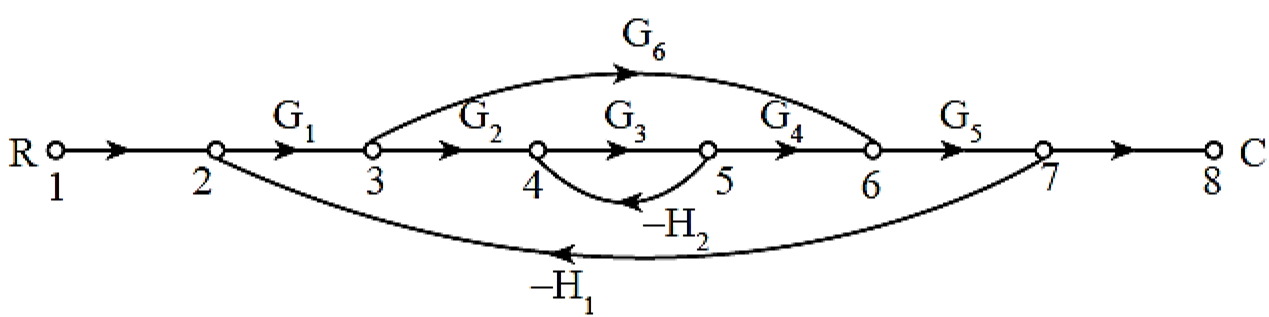
Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

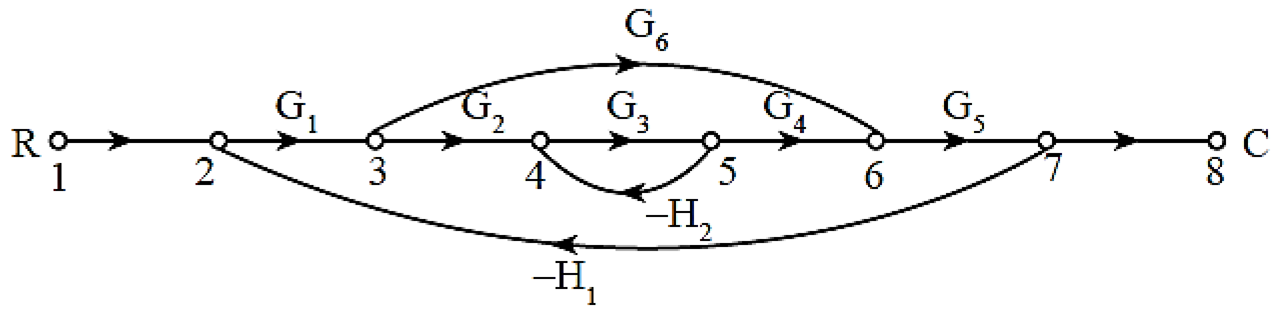
Δ_k = value of Δ for the part of the block diagram that does not touch the k-th forward path ($\Delta_k = 1$ if there are no non-touching loops to the k-th path.)

Example 1 : Determine the control ratio of a system represented by signal flow graph as shown in figure using Mason's gain formula.



First number the nodes



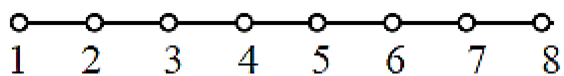


Step 1 :- Find forward path and forward path gain

Forward path :-

Forward path gain:-

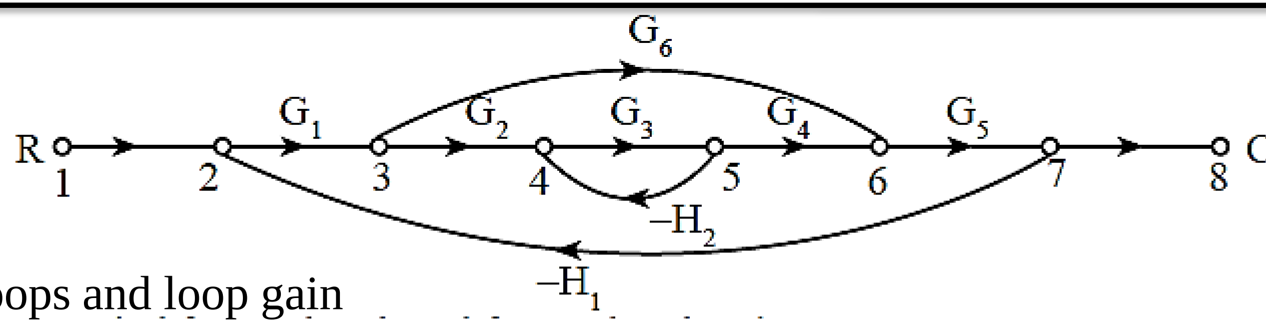
(1)



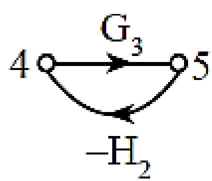
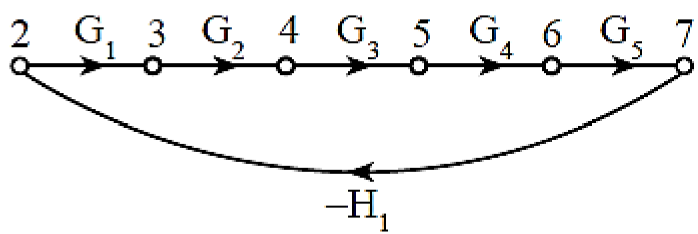
$P_1 = G_1G_2G_3G_4G_5$

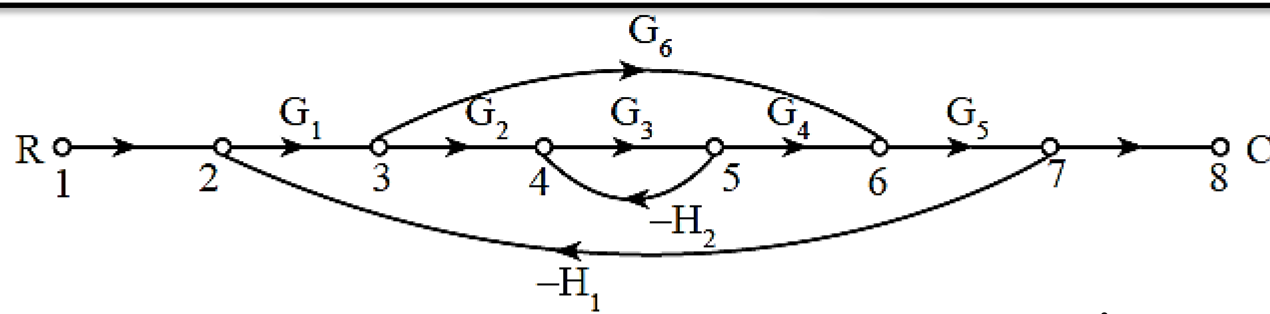
(2)

$P_2 = G_1G_5G_6$



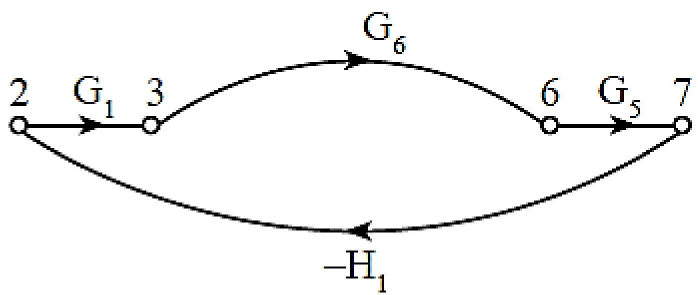
Step 2 :- Find loops and loop gain

Loops		**Loops gain**
(1)		$L_1 = -G_3H_2$
(2)		$L_2 = -G_1G_2G_3G_4G_5H_1$



Loops **Loops gain**

(3)



$L_3 = -G_1G_5G_6H_1$

Step 3 :- Find Non touching loops (NTL)

$L_1L_3 = G_1G_3G_5G_6H_1H_2$

$$\text{Mason gain formula}, \frac{C}{R} = \frac{p_1\Delta_1 + p_2\Delta_2}{\Delta}$$

Step 4 :- Find Determinant (D) :

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3] - [\cancelto{0}{\;}]$$

$$= 1 - [-G_3H_2 - G_1G_2G_3G_4G_5H_1 - G_1G_5G_6H_1] + [G_1G_3G_5G_6H_1H_2]$$

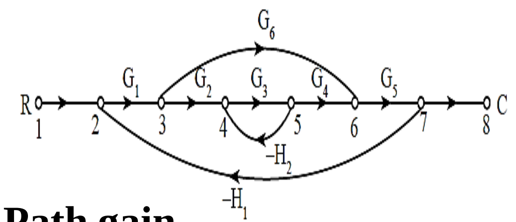
$$\boxed{\Delta = 1 + G_3H_2 + G_1G_2G_3G_4G_5H_1 + G_1G_5G_6H_1 + G_1G_3G_5G_6H_1H_2}$$

$$\Delta_1 = 1 - [\cancelto{0}{\;}] + [\cancelto{0}{\;}]$$

$$\boxed{\Delta_1 = 1}$$

$$\Delta_2 = 1 - [L_1] + [\cancelto{0}{\;}]$$

$$\boxed{\Delta_2 = 1 + G_3H_2}$$



Path gain

$P_1 = G_1G_2G_3G_4G_5$

$P_2 = G_1G_5G_6$

Loop gain

$L_1 = -G_3H_2$

$L_2 = -G_1G_2G_3G_4G_5H_1$

$L_3 = -G_1G_5G_6H_1$

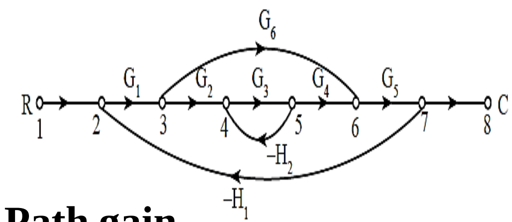
NTL:

$L_1L_3 = G_1G_3G_5G_6H_1H_2$

Mason gain formula, $\frac{C}{R} = \frac{p_1\Delta_1 + p_2\Delta_2}{\Delta}$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4G_5 + G_1G_6G_5(1+G_3H_2)}{1+G_3H_2+G_1G_2G_3G_4G_5+G_4G_5G_6H_1+G_1G_3G_5G_6H_1H_2}$$

$$\boxed{\frac{C}{R} = \frac{G_1G_2G_3G_4G_5 + G_1G_6G_5 + G_1G_3G_5G_6H_2}{1+G_3H_2+G_1G_2G_3G_4G_5+G_4G_5G_6H_1+G_1G_3G_5G_6H_1H_2}}$$



Path gain

$P_1 = G_1G_2G_3G_4G_5$

$P_2 = G_1G_5G_6$

Loop gain

$L_1 = -G_3H_2$

$L_2 = -G_1G_2G_3G_4G_5H_1$

$L_3 = -G_1G_5G_6H_1$

NTL:

$L_1L_3 = G_1G_3G_5G_6H_1H_2$

thank you