CONTROL ENGINEERING 18ME71

Module - 3 Steady State Operation & Transient Response

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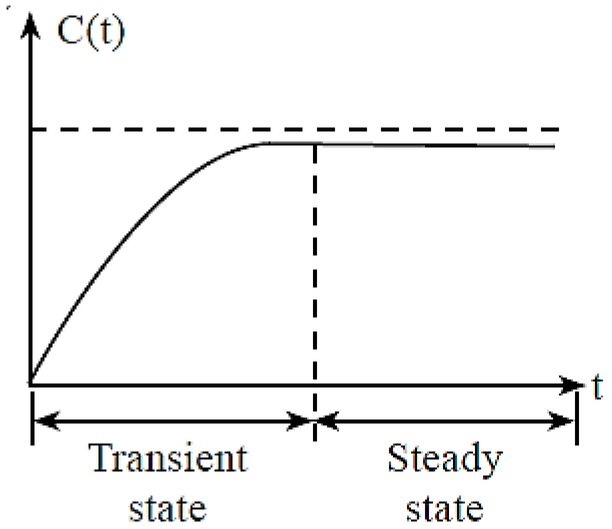
OBJECTIVES:

- To illustrate Transient and steady state response analysis of a system.
- To analyse the system using root locus plots.
- To study different system compensators used in Control system.
- Determine the stability of control system Frequency response analysis using Root locus plot
- To study the characteristic behavior of Compensators.

Time response

- Transient response
- Steady state response



$$\mathbf{C(t) = C_{tr}(t) + C_{ss}(t)}$$

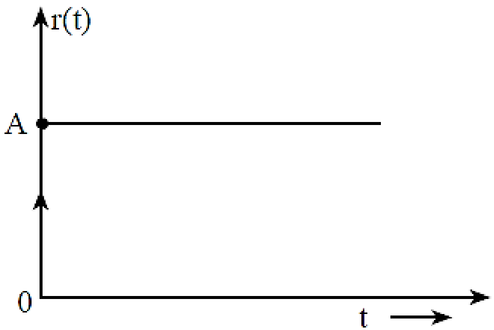
Where,

$C_{tr}(t)$ = Transient response

$C_{ss}(t)$ = Steadystate response

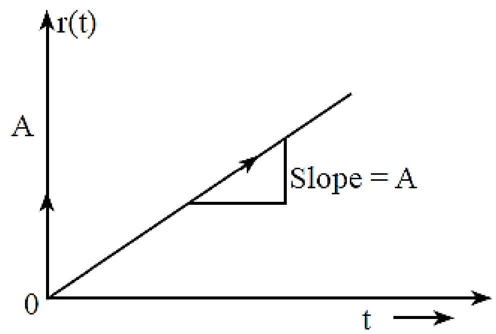
Standard test inputs :

1. Step input :



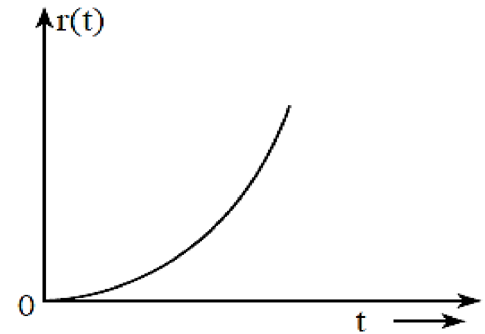
$$r(t) = A \quad \text{at } t > 0$$
$$r(t) = 0 \quad \text{at } t < 0$$

2. Ramp Input :



$$r(t) = At \quad \text{at} \quad t > 0$$
$$r(t) = 0 \quad \text{at} \quad t < 0$$

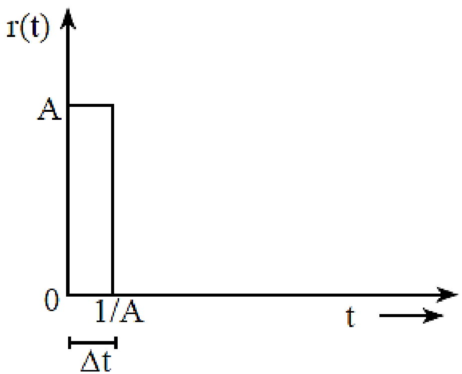
3. Parabolic input :



$$r(t) = A\frac{t^2}{2} \quad \text{at} \quad t > 0$$
$$r(t) = 0 \quad \text{at} \quad t < 0$$

Standard test inputs :

4. Impulse input :



$$r(t) = A \quad \text{at} \quad t \geq 0$$
$$r(t) = 0 \quad \text{at} \quad t \neq 0$$

❖ Steady state analysis for general block diagram of control system

1. Time taken by a system to reach its steady state known as "***settling time***".
2. Error occurred between actual output and derived output known as "***steady state error(e_{ss})***".

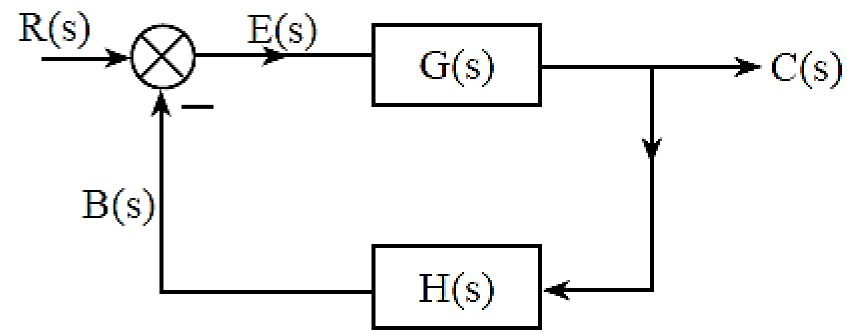
$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - C(s)H(s)$$

$$E(s) = R(s) - E(s)G(s)H(s)$$

$$E(s) + E(s)G(s)H(s) = R(s)$$

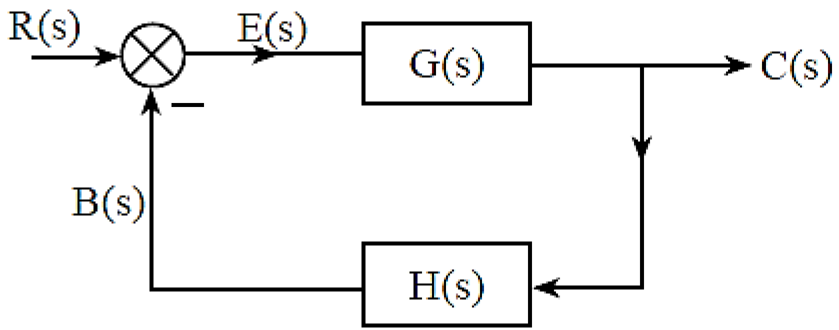
$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$



For a unity feedback system H(s) = 1

$$\boxed{E(s) = \frac{R(s)}{1 + G(s)}}$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$



WKT, Steady state error, $e_{ss} = \lim_{t \to \infty} e(t)$

By relating final value theorem

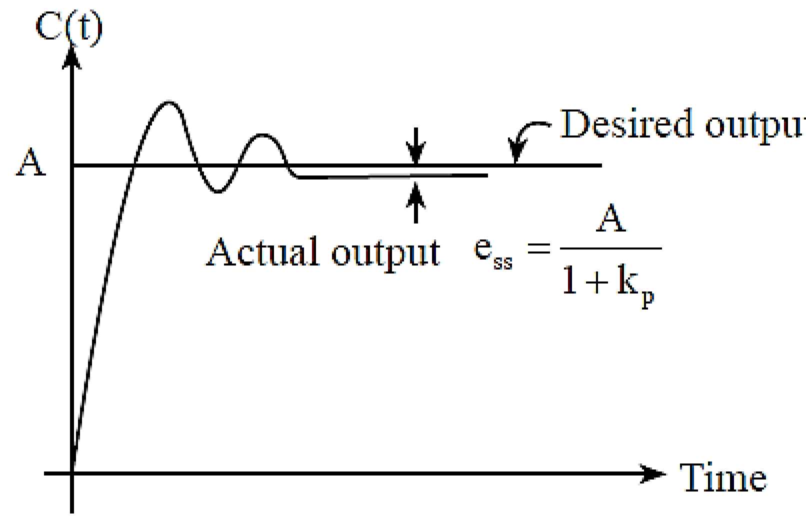
$$\lim_{t \to \infty} E(t) = \lim_{s \to 0} sE(s) \quad \Rightarrow \quad e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

Steady state error for a closed loop system depends on,

1. Reference input i.e., R(s)
2. Open tool transfer function i.e., G(s) H(s)

Steady state characteristics

1. Step input with magnitude 'A'



We know that step input

$$R(s) = \frac{A}{S}$$

$$\therefore e_{ss} = \lim_{S \to 0} \frac{SR(s)}{1+G(s)H(s)}$$

$$= \lim_{S \to 0} \frac{S \times \cancel{A}/\cancel{s}}{1+G(s)H(s)}$$

$$= \lim_{S \to 0} \frac{A}{1+G(s)H(s)}$$

$$\boxed{e_{ss} = \frac{A}{1+k_p}}$$

Steady state characteristics

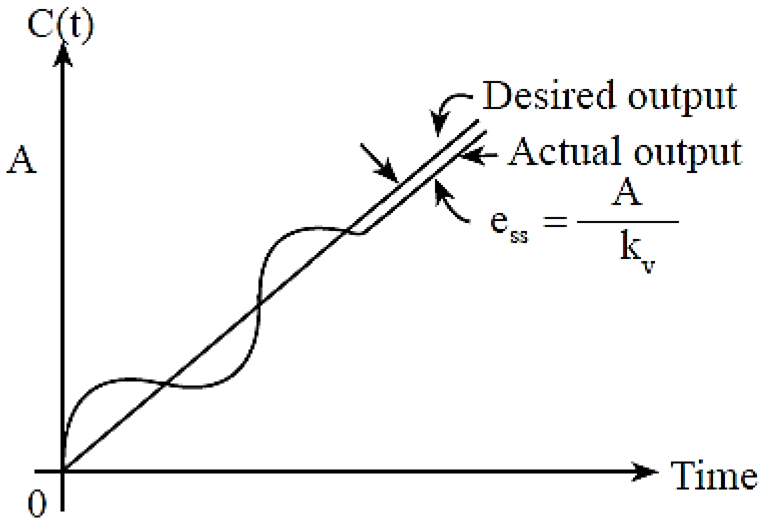
2. Ramp input of magnitude 'n' :

$$R(s) = A/_{S^2}$$

$$e_{ss} = \lim_{S\to 0} \frac{S \times A/_{S^2}}{1+G(s)H(s)}$$

$$= \lim_{S\to 0} \frac{A}{S+S[G(s)H(s)]}$$

$$\boxed{\therefore e_{ss} = \frac{A}{k_v}}$$



Steady state characteristics

Type of input	Steady state error	Error coefficient
Step	$e_{ss} = \frac{A}{1+k_p}$	$k_p = \lim_{x \to 0} G(s)H(s)$
Ramp	$e_{ss} = \frac{A}{k_v}$	$k_v = \lim_{s \to 0} S\left[G(s)H(s)\right]$
Parabolic	$e_{ss} = \frac{A}{k_a}$	$k_a = \lim_{S \to 0} S^2\left[G(s)H(s)\right]$

Transient response specifications

1. Delay time (T_d) :

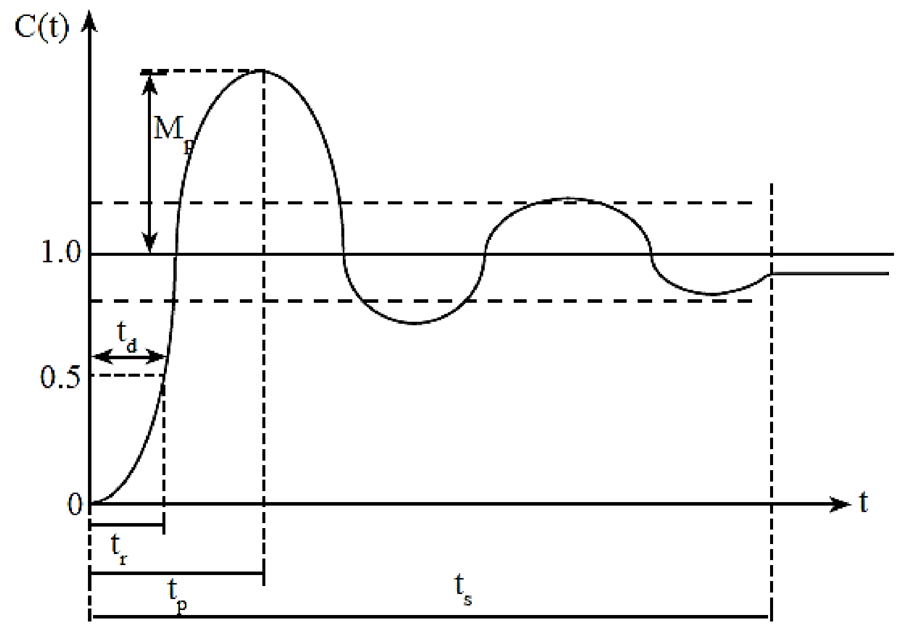
$$\boxed{T_d = \frac{1+0.7\xi}{\omega_n}}$$

2. Rise time (T_r) :

$$\boxed{T_r = \frac{\pi-\theta}{\omega_d}\sec}$$

3. Peak time (T_p) :

$$\boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}\ \sec}$$



Transient response specifications

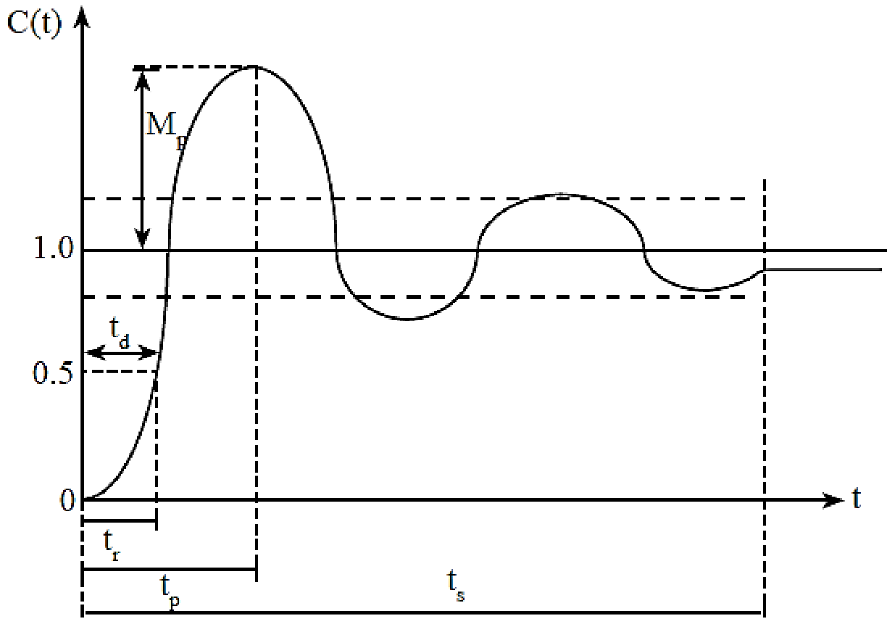
4. Peak overshoot (M_p) :

$$\boxed{M_p = e^{-\pi\xi / \sqrt{1-\xi^2}}}$$

5. Setting time (T_s) :

$$\boxed{T_s = 2 \text{ or } 4 \times \text{Time constant}}$$

$$\text{Time constant, } \boxed{T = \frac{1}{\xi\omega_n}}$$



Example 1: A unity feedback characterized by an open loop transfer function

$$G(s) = \frac{10}{S^2 + 2S + 6}$$

Determine, 1. Undamped natural frequency,
2. Damped ratio 3. Damped Frequency
4. Peak time 5. Settling time

To obtained characteristic equation we have,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{10}{s^2 + 2s + 6} \times 1 = 0$$

$$\frac{s^2 + 2s + 6 + 10}{s^2 + 2s + 6} = 0$$

$$\boxed{s^2 + 2s + 16 = 0}$$ **--Eq (1)**

$$s^2 + 2s + 16 = 0$$ **--Eq (1)**

We Know Standard Characteristic equation,

$$s^2 + 2\xi\omega_n s + {\omega_n}^2 = 0$$ **--Eq (2)**

$${\omega_n}^2 = 16 \Rightarrow \boxed{\omega_n = 4 \text{ rad/s}}$$ ***Undamped natural frequency***

2. Damped ratio

$$2\xi\omega_n s = 2s$$

$$\xi = \frac{2}{2\omega_n} = \frac{1}{4}$$

$$\boxed{\xi = 0.25}$$

3. Damped frequency

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = 4\sqrt{1-0.25^2}$$

$$\boxed{\omega_d = 3.872 \text{ rad/s}}$$

4. Peak time

$$T_p = \frac{\pi}{\omega_d}$$

$$T_p = \frac{\pi}{3.872}$$

$$\boxed{T_p = 0.811 \text{ sec}}$$

5. Settling time

$$T_s = \frac{4}{\xi \times \omega_n}$$

$$T_s = \frac{4}{0.25 \times 4}$$

$$\boxed{T_s = 4 \text{ sec}}$$

Routh Stability criteria

E.J Routh and ***A. Hurwitz*** individually found the method for investigating the stability of a system.

Let the characteristic equation of a nth order system as follows.

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots\ldots.\ a_n s^0 = 0$$

Example 1 : Determine the stability of a system whose characteristic equation is given by

$S^4 + 5S^3 + 20S^2 + 40S + 50 = 0$, using Routh criteria.

Solution.

s^4	1	20	50
s^3	5	40	0
s^2	12	50	
s^1	19.6		
s^0	50		

$$\frac{(20\times 5)-(40\times 1)}{5}=12$$

$$\frac{(5\times 50)-0}{5}=50$$

$$\frac{(12\times 40)-(5\times 50)}{12}=19.6$$

$$\frac{(13.6\times 50)-0}{13.6}=50$$

Root locus plots

Root locus technique is a graphical method of plotting the locus of roots of characteristic equation in the "S -plane" as a system parameter "k" is varied.

This method was developed by "Evan" in 1948. It is a way of presenting graphical information about system's behavior.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \Rightarrow \quad G(s)H(s) = \frac{k(s+z_1)(s+z_2)\ldots(s+z_n)}{(s+P_1)(s+P_2)\ldots(s+P_m)}$$

Z Number of Zero's
P Number of poles
K Scalar gain

Rules for constructing locus plot :

1. The root locus begins at open loop poles (p):

$\times$

2. The root locus terminates at open loop Zero's (Z):

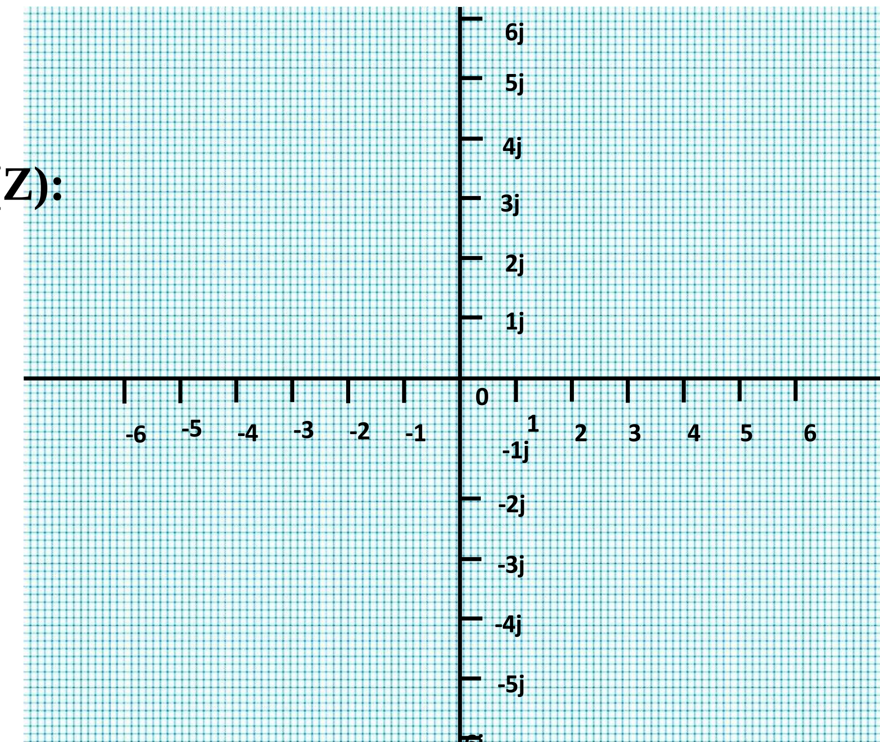
$\otimes$

3. Number of branches (N) :

N = P when P > Z
N = Z when Z > P

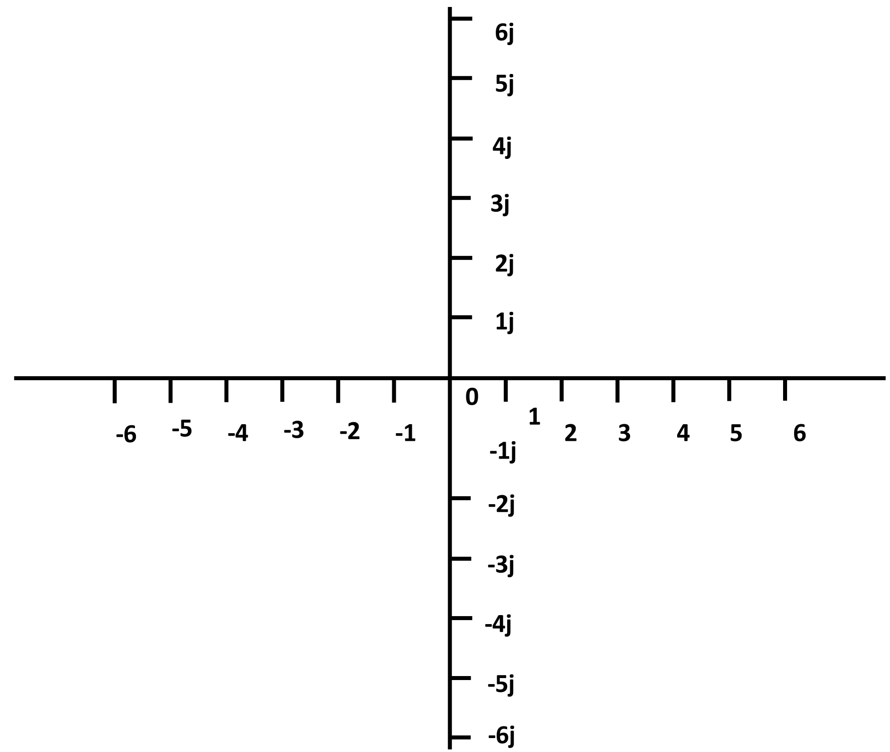
4. Number of branches terminating at infinity :

P – Z



5. Root locus is symmetrical about the real axis:

6. The root - locus lies on the real axis :



7. Angle of asymptotes (θ_K) :

$$\theta_k = \frac{(2k+1)180^0}{P-Z}$$

$$k = 0,1,2 \ldots .(P-Z-1)$$

8. Centroid (σ) :

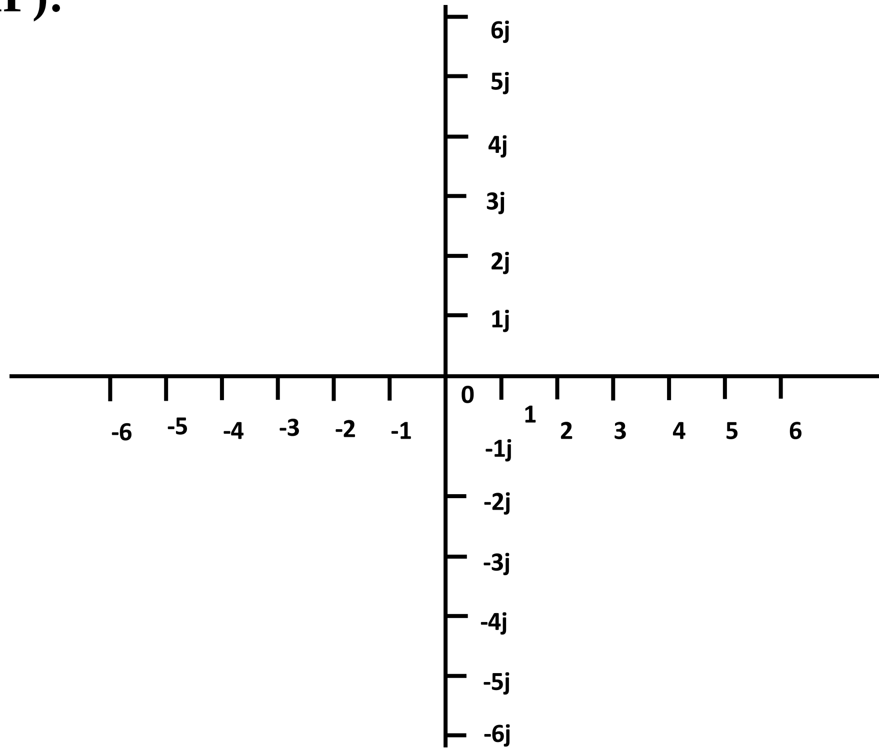
$$\sigma = \frac{\sum P - \sum Z}{P-Z}$$

9. Break away point (BAP) and break in point (BIP):

10. Cross-over point of the root - locus with the imaginary axis :

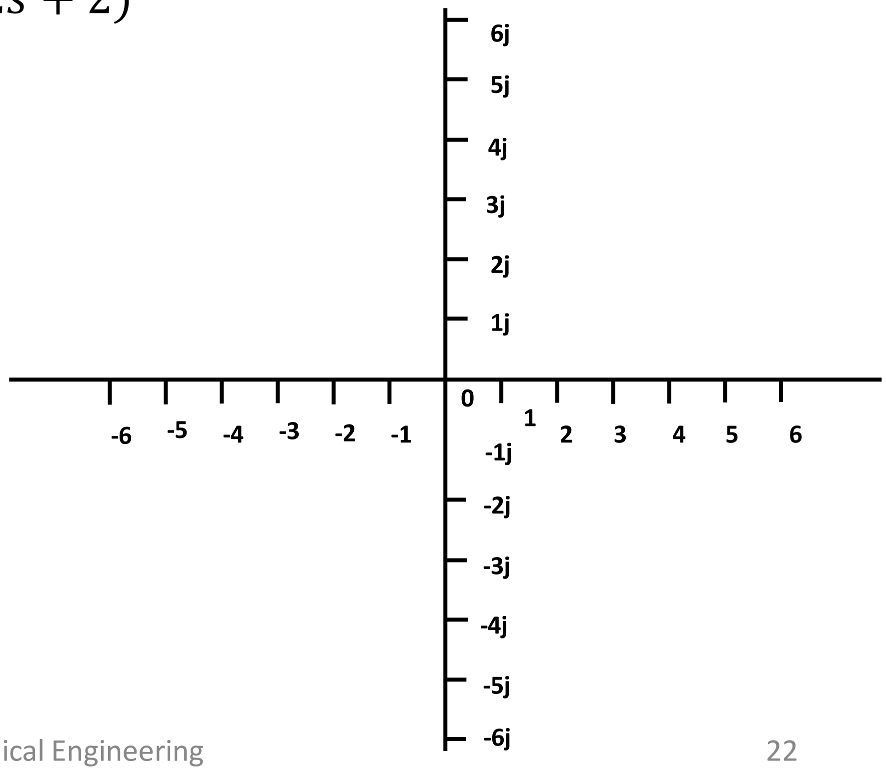
11. Angle of departure:

12. Angle of arrival :



Example 4 : Plot root - locus for the following function

$$G(s)H(s) = \frac{k}{s(s+3)(s^2+2s+2)}$$



Solution:

1. Root locus begins at open loop poles.

P = 4

$S_1 = 0$, $S_2 = -3$, $S_3 = -1+j$, $S_4 = -1-j$

2. Root locus terminates at open loop zeros

Z = 0

3. Number of branches (N) :

P > Z N = P = 4

4. Number of branches terminating at infinity (∞) :

$$P - Z$$

$$4 - 0 \Rightarrow 4$$

5. Root locus is symmetrical about the real axis.

6. Root locus branch present on the real axis.

7. Angle of asymptotes (θ_K) :

$$\theta_k = \frac{(2k+1)180^0}{P-Z}$$

$$k = 0,1,2 \dots .(P-Z-1)$$
$$k = 0,1,2 \dots .(4-0-1)$$
$$k = 0,1,2,3$$

$$\theta_0 = \frac{(2\times 0+1)180^0}{4-0}$$
$$\theta_0 = 45^0$$

$\theta_1 = 135^0$ $\quad\theta_2 = 225^0$ $\quad\theta_3 = 315^0$

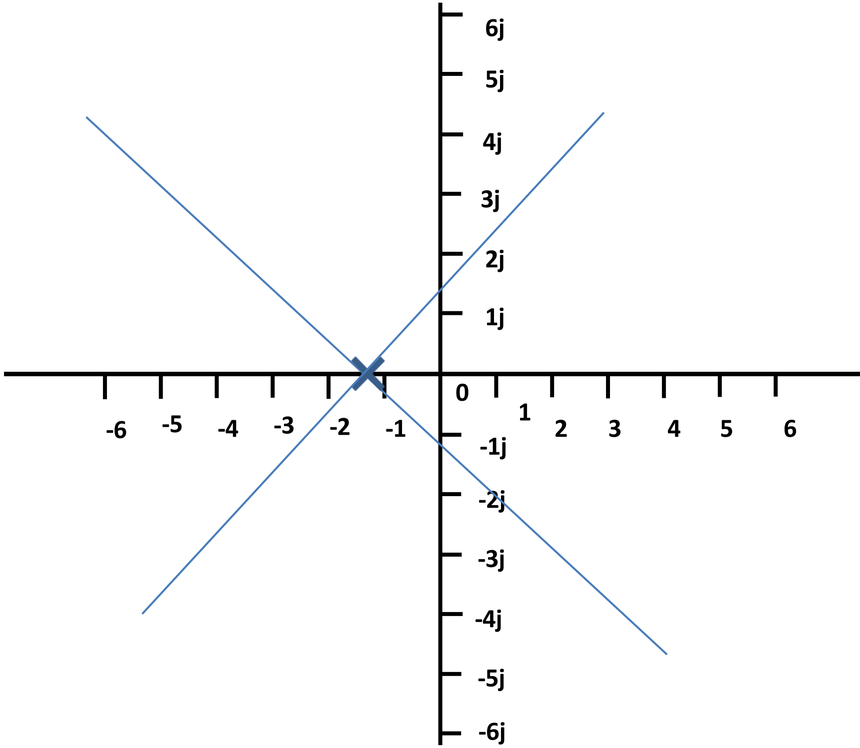
8. Centroid (σ) :

$$\sigma = \frac{\sum P - \sum Z}{P - Z}$$

$S_1 = 0$, $S_2 = -3$, $S_3 = -1+j$, $S_4 = -1-j$

$$\sigma = \frac{(0 - 3 - 1 + j - 1 - j) - (0)}{4 - 0}$$

$$\sigma = -1.25$$



9. Break away point :

$$G(s)H(s) = \frac{k}{s(s+3)(s^2+2s+2)}$$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k}{s(s+3)(s^2+2s+2)} = 0$$

$$1 + \frac{k}{(s^2+3s)(s^2+2s+2)} = 0$$

$$(s^2+3s)(s^2+2s+2) + k = 0$$

$$\boxed{s^4 + 5s^3 + 8s^2 + 6s + k = 0}$$ Characteristic equation

$$k = -(s^4 + 5s^3 + 8s^2 + 6s)$$

$$\frac{dk}{ds} = 0 \Rightarrow 4s^3 + 15s^2 + 16s + 6 = 0 \Rightarrow$$

$s_1 = -2.29$

$s_2 = -0.73+0.35j$

$s_3 = -0.73-0.35j$

10. Cross-over point of the root locus with the imaginary axis :

$$s^4 + 5s^3 + 8s^2 + 6s + k = 0$$

s^4	1	8	k
s^3	5	6	
s^2	6.8	k	
s^1	$\frac{40.8 - 5k}{6.8}$		
s^0	k		

For stability condition

$$k \geq 0$$

$$\frac{40.8 - 5k}{6.8} \geq 0$$

$$k = 8.16$$

$$0 \leq k \leq 8.16$$

Considering "S^2" terms

$$6.8s^2 + k = 0$$

$$6.8s^2 + 8.16 = 0$$

$$6.8s^2 = -8.16$$

$$s = \pm 1.1j$$

11. Angle of Departure:

$$\emptyset_{P1} = 90^0$$

$$\emptyset_{P2} = 180^0 - \tan^{-1}\left(\frac{1}{1}\right) = 135^0$$

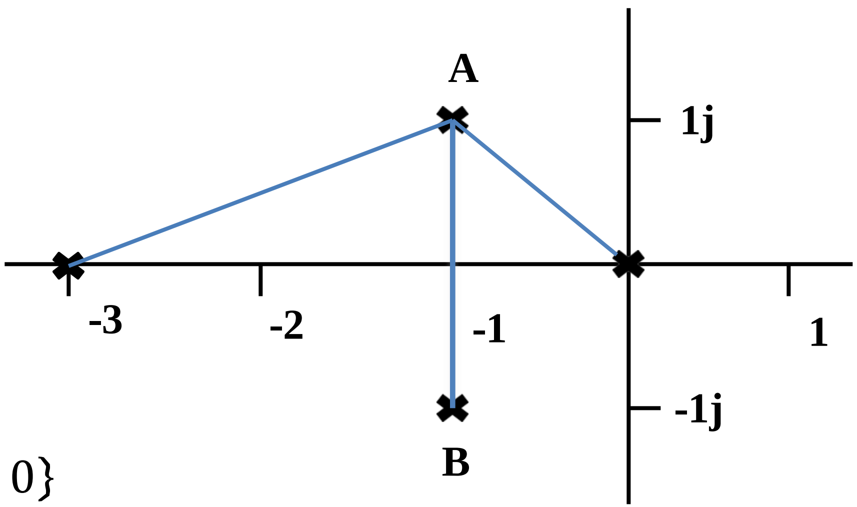
$$\emptyset_{P3} = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^0$$

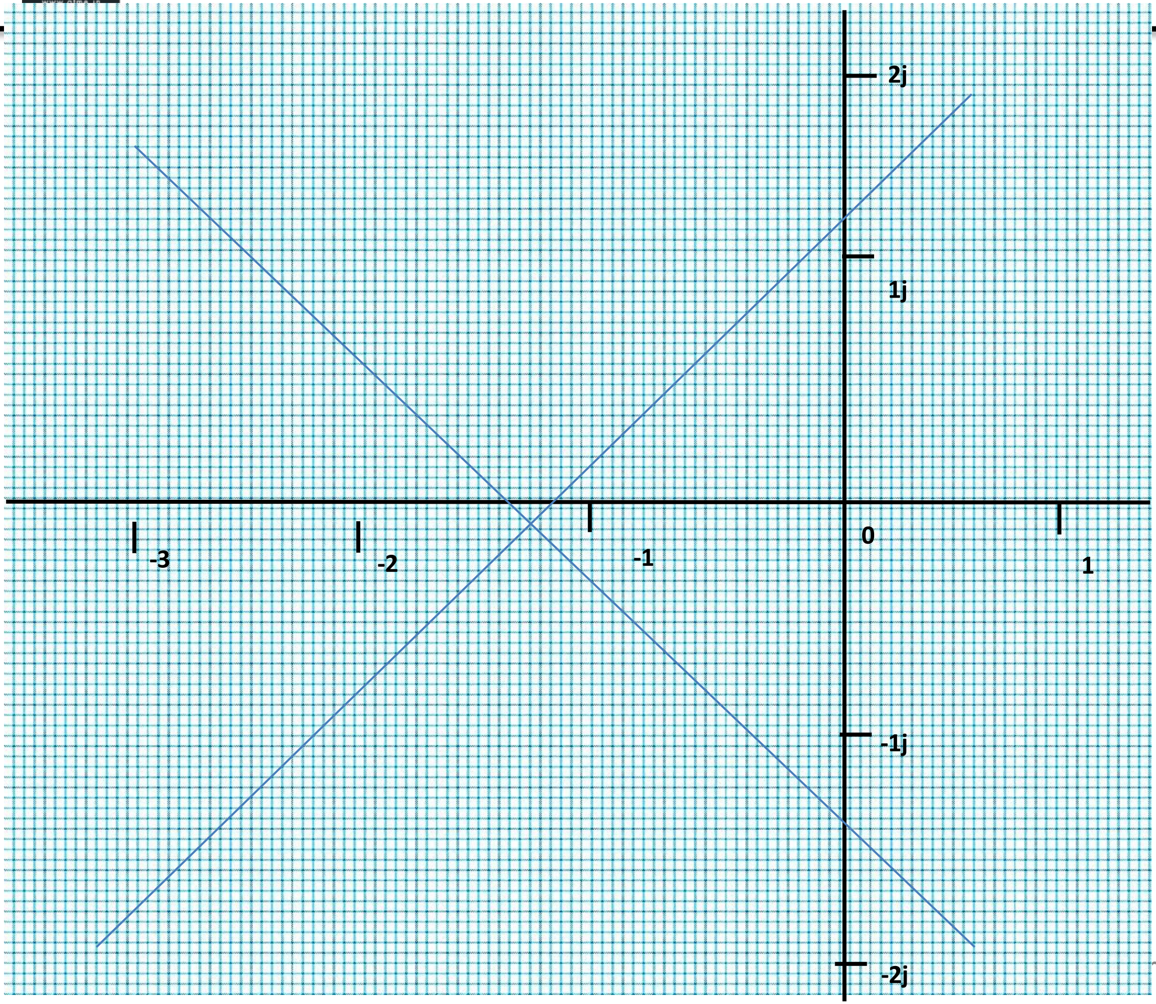
$$\theta_A = 180^0 - \left\{\sum \emptyset_P - \sum \emptyset_Z\right\}$$

$$\theta_A = 180^0 - \{90^0 + 135^0 + 26.56^0 - 0\}$$

$$\theta_A = -71.56^0$$

$$\theta_B = -\theta_A = 71.56^0$$





P = 4

$S_1 = 0,\ S_2 = -3,\ S_3 = -1+j,\ S_4 = -1-j$

$$\boldsymbol{\sigma = -1.25}$$

$$Asymptotes:$$
$$45^0, 135^0, 225^0, 315^0$$

BAP

s_1=-2.29

s_2=-0.73+0.35j

s_3=-0.73-0.35j

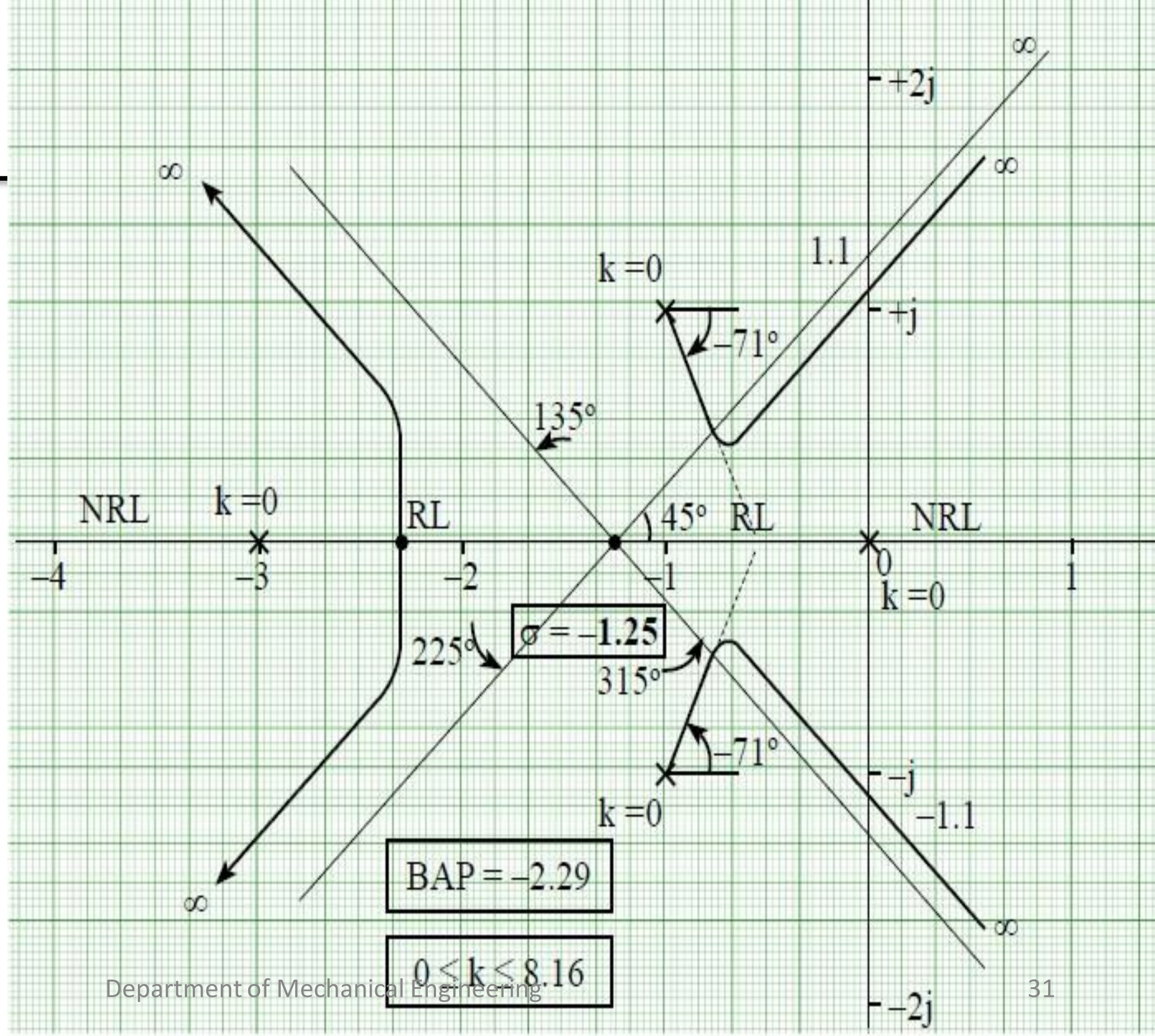
$$\omega = \pm 1.1j$$

$$\theta_A = -71.56^0$$
$$\theta_B = 71.56^0$$

Comment :

The value of "k" ranges between 0 and 8.16 the system will be stable up to k = 8.16 and becomes unstable for the values greater than 8.16.

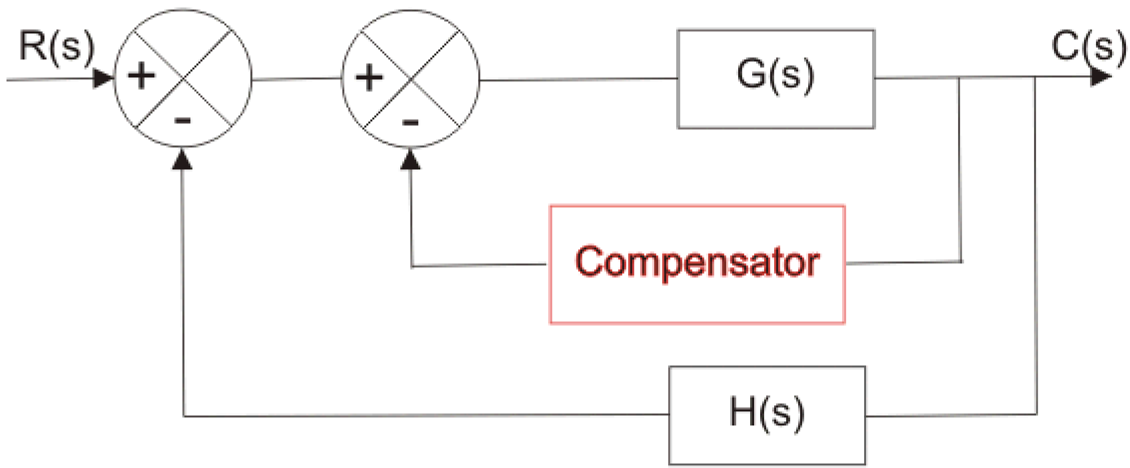
The frequency of the sustained oscillation is, ω = 1.1 rad/s



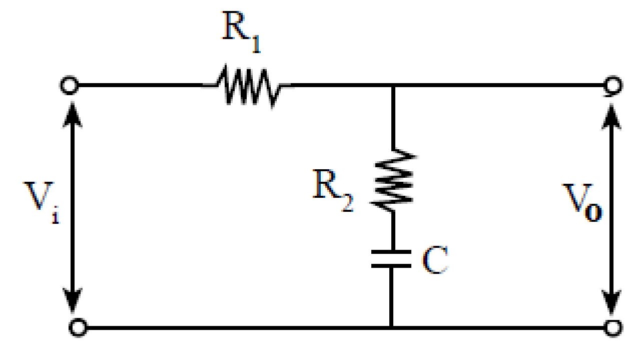
Compensator

The additional component or device compensates the performance deficiency and is called as compensator.

The process of redesign at addition of device is called as compensation.



Lag compensator :



The transfer function of this compensator is given by

$$\frac{V_0}{V_i} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$

Substituting $R_2 C = r$ $\qquad \frac{R_1 + R_2}{R_2} = \alpha$

$$\frac{V_0}{V_i} = \frac{rs + 1}{r\alpha s + 1}$$

$$\frac{V_0}{V_i} = \frac{1}{\alpha}\left[\frac{s + \frac{1}{r}}{s + \frac{1}{r\alpha}}\right]$$

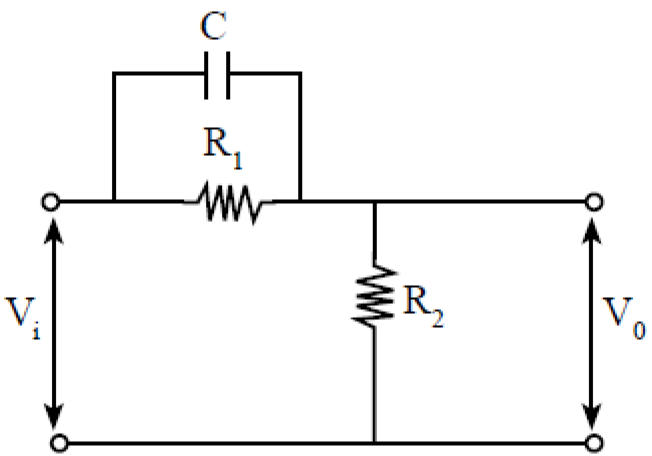
Disadvantages of lag compensator :

1. The system transient response will have a slow term
2. Reduced bandwidth is disadvantage in some system.
3. Due to the presence of phase lag compensation the speed of the system decreases.

Advantages of lag compensator

1. Low frequency characteristics are improved i.e., it permits accuracy high gain at low frequencies, which improves steady state accuracy and reduces gain in high range of frequencies.
2. Stability margins are improved.
3. Due to the presence of phase lag compensation the steady state accuracy increases.

Lead compensation :



Transfer function of electrical phase - lead network is given by

$$\frac{V_0}{V_i} = \left(\frac{R_2}{R_1+R_2}\right)\frac{R_1Cs+1}{\left(\frac{R_1R_2}{R_1+R_2}\right)Cs+1}$$

Substituting $R_1C = r$ $\quad \frac{R_2}{R_1+R_2} = \alpha$

$$\frac{V_0}{V_i} = \alpha\,\frac{rs+1}{\alpha rs+1}$$

$$\frac{V_0}{V_i} = \left[\frac{s+\frac{1}{r}}{s+\frac{1}{r\alpha}}\right]$$

Advantages of lead compensator :

1. The lead compensator are basically high pass filter. The high frequencies are passed but low frequencies are attenuator
2. Improves high frequency performance such as speed of response
3. Lead compensator increases the bandwidth.
4. Due to the presence of phase lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
5. Due to the presence of phase lead compensation maximum overshoot of the system decreases.

Disadvantages of lead compensator :

1. Accentuated high frequency noise problems.
2. May generate layer signals which may damage the system or cause non-linear operation of the system.
3. Steady state error is not improved

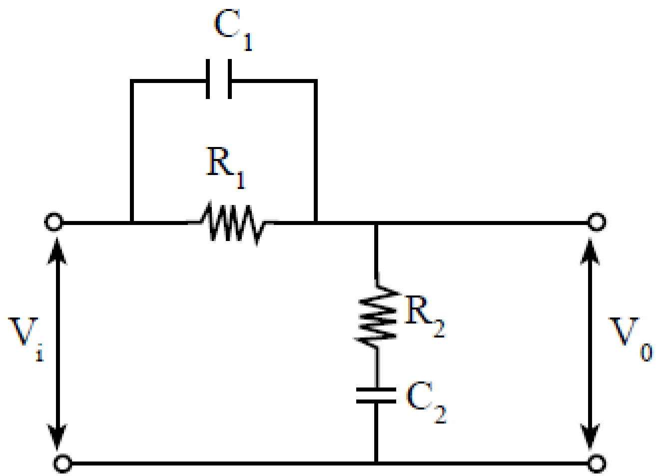
Comparison between lead and lag compensators

Lead compensator	Lag compensator
High pass	Low pass
Approximates derivative plus proportional control	Approximates integral plus proportional control
Contributes phase lead	Attenuation at high frequencies
Increases the gain crossover frequency	Moves the gain-crossover frequency lower
Increases bandwidth	Reduces bandwidth

Lag-lead compensator :

Transfer function is given by

$$\frac{V_0}{V_i} = \frac{(R_1C_1s + 1)(R_2C_2s + 1)}{(R_1C_1s + 1)(R_2C_2s + 1) + R_1C_2s}$$



Substituting

$$R_1C_1 = r_1 \qquad R_2C_2 = r_2 \qquad \frac{R_2}{R_1 + R_2} = \alpha$$

$$\frac{V_0}{V_i} = \frac{\left(s + \frac{1}{r_1}\right)\left(s + \frac{1}{r_2}\right)}{\left(s + \frac{\alpha}{r_1}\right)\left(s + \frac{1}{\alpha r_2}\right)}$$

Advantages of lag - lead compensator :

1. Lag - lead compensator combines the lag-lead combinations
2. Improves the steady state accuracy of the system.
3. It increases the system bandwidth to achieve faster response of the system.
4. Due to the presence of phase lag-lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.

Previous year Question Paper

DEC18-JAN19

Model Question Paper 1

Model Question Paper 2

thank you