



A T M E

College of Engineering

Department of Mechanical Engineering



CONTROL ENGINEERING 18ME71

Module - 4 Frequency Domain Analysis

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OBJECTIVES:

- To study the Frequency response analysis using polar plot, Nyquist plot and Bode plot.

Introduction :

Why Frequency Response?

1. Weakness of root locus method relies on the existence of open-loop transfer function
2. Weakness of time-domain analysis method is that time response is very difficult to obtain
 - Computational complex
 - Difficult for higher order system
 - Difficult to partition into main parts
 - Not easy to show the effects by graphical method

Advantages of Frequency Response Analysis:

- Frequency response (mathematical modeling) can be obtained directly by experimental approaches.
- Easy to analyze effects of the system with sinusoidal signals.
- Convenient to measure system sensitivity to noise and parameter variations.

Note: *Frequency domain analysis* is a kind of (indirect method) engineering method. It studies the system based on frequency response which is also a kind of mathematical model.

Frequency response (or characteristic) is the ratio of the complex vector of the steady-state output versus sinusoidal input for a linear system.

$$G(j\omega) = \frac{1}{1 + j\omega T} = A(\omega)e^{j\phi(\omega)}$$

$$A(\omega) = \left| \frac{1}{1 + j\omega T} \right| = \frac{1}{\sqrt{1 + \omega^2 T^2}}; \quad \text{Magnitude response}$$

$$\phi(\omega) = \angle\left(\frac{1}{1 + j\omega T}\right) = -\arctg\omega T \quad \text{Phase response}$$

$\omega = 0$ The output has same magnitude and phase with input

$\omega \uparrow$ Magnitude will be attenuated and phase lag is increased

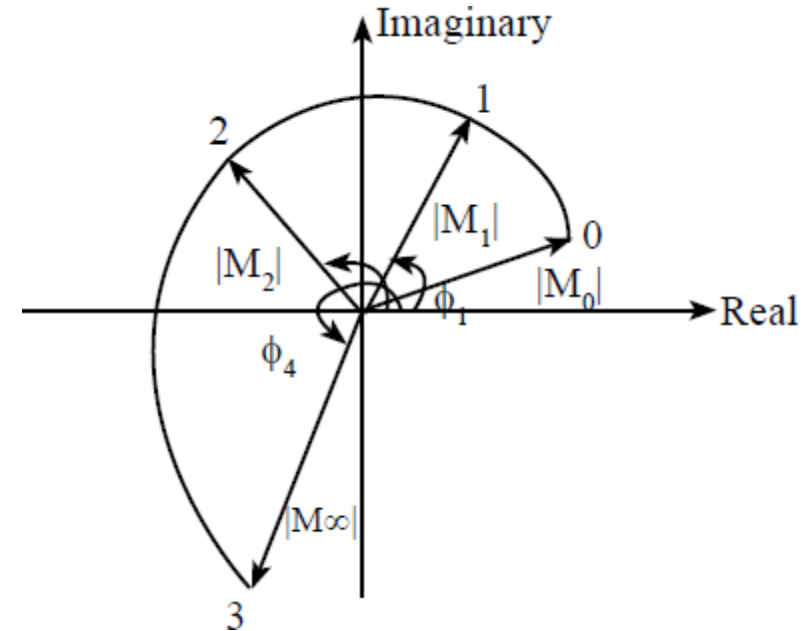
Polar plot :

Plotting the variations of magnitude and phase angle v/s input frequency

$$\text{Magnitude, } M = |G(j\omega)H(j\omega)|$$

$$\text{Phase angle, } \phi = \angle G(j\omega)H(j\omega)$$

It can be calculated the value of **Magnitude (M)**
and **Phase angle (Ø)** by varying the value of " ω "
from "**0**" to " **∞** "



Example 1: Obtain polar plot for a open loop transfer function.

$$G(s)H(s) = \frac{20}{s}$$

Solution:

Replace 'S,, by 'j ω '




$$G(j\omega)H(j\omega) = \frac{20}{j\omega}$$

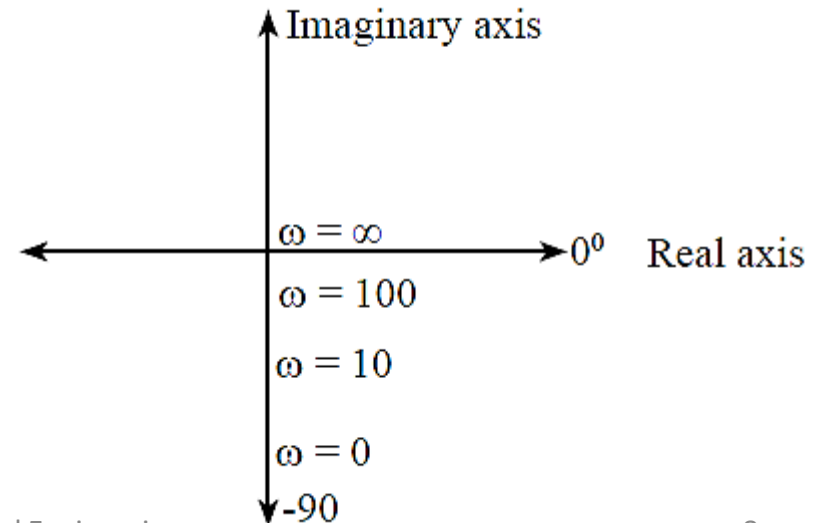
$$\text{Magnitude, } M = |G(j\omega)H(j\omega)| = \frac{20}{\omega}$$

$$\text{Phase angle, } \phi = \angle G(j\omega)H(j\omega) = \frac{\tan^{-1}\left(\frac{0}{20}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)} = \frac{0^\circ}{\tan^{-1}\left(\frac{\omega}{0}\right)}$$

$$\text{Magnitude, } M = |G(j\omega)H(j\omega)| = \frac{20}{\omega}$$

$$\text{Phase angle, } \phi = \angle G(j\omega)H(j\omega) = \frac{0^\circ}{\tan^{-1}\left(\frac{\omega}{0}\right)} = \frac{0^\circ}{90^\circ}$$

ω	Magnitude (M)	Phase angle (ϕ)
0	∞	-90°
10	2	-90°
		
∞	0	-90°



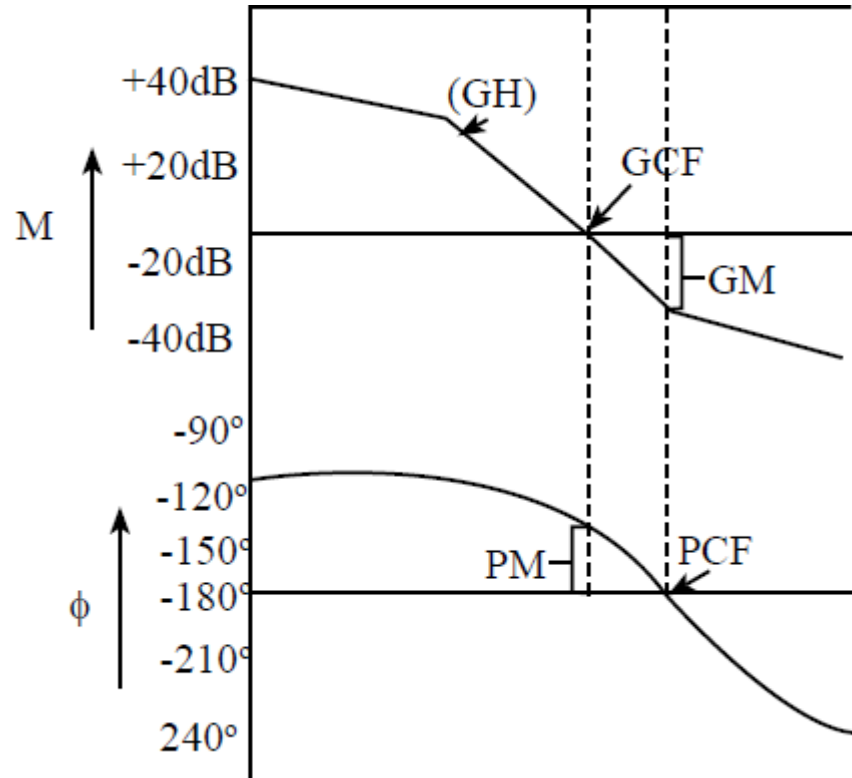
Code plots

H.W Bode suggested the method in which logarithmic values of magnitude is to be plotted against logarithmic values of frequency, such plots are called “*logarithmic plots*”.

Bode diagram consists of two plots,

1. Magnitude expressed in logarithmic values of frequency called as **Magnitude plot**.
2. Phase angle in degrees against logarithmic values of frequency called as **Phase plot**.

	Magnitude (dB)	Slope (dB/decade)	Phase angle (degrees)	Corner frequency (rad/sec)
k	$M = 20 \log_{10} k$	0	$\phi = 0^\circ$ if $k = +ve$ OR $\phi = 180^\circ$ if $k = -ve$	Nil
$S^{\pm n}$	$M = \pm n 20 \log_{10} \omega$	$\pm n 20$	$\phi = \pm n 90^\circ$	Nil
$(1+ST)^{\pm n}$	$M = \pm n 20 \log_{10} \sqrt{1 + (\omega T)^2}$	$\pm n 20$	$\phi = \pm n \tan^{-1} \left(\frac{\omega T}{1} \right)$	$\omega_{CT} = \frac{1}{T}$
$(s^2 + 2\xi\omega_n s + \omega_n^2)^{\pm n}$	$M = \pm n 40 \log_{10} \left(\frac{\omega}{\omega_n} \right)$	$\pm n 40$	$\phi = \pm n \tan^{-1} \left(\frac{2\xi \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n} \right)^2} \right)$	$\omega_{CF} = \omega_n$



GM = Gain Margin

PM = Phase Margin

GCF = Gain crossover frequency

PCF = Phase cross over frequency

Example 1: The open loop transfer function of a system is given by $G(s)H(s) = \frac{10(s+10)}{s(s+2)(s+5)}$

Draw bode diagram. Find gain cross - over frequency (GCF), phase cross-over frequency (PCF), gain margin (GM) phase margin (PM). Find stability of the system.

Solution

Step1:

$$G(s)H(s) = \frac{10 \times 10(1 + 0.1s)}{s \times 2(1 + 0.5s) \times 5 \times (1 + 0.2s)}$$

$$G(s)H(s) = \frac{10(1 + 0.1s)}{s(1 + 0.5s)(1 + 0.2s)}$$

$$G(s)H(s) = \frac{10(1 + 0.1s)}{s(1 + 0.5s)(1 + 0.2s)}$$

Step 2: Replace „S“ by “jω”

$$G(j\omega)H(j\omega) = \frac{10(1 + 0.1j\omega)}{(j\omega)(1 + 0.5j\omega)(1 + 0.2j\omega)}$$

$$G(j\omega)H(j\omega) = \frac{10(1 + 0.1j\omega)}{(j\omega)(1 + 0.5j\omega)(1 + 0.2j\omega)}$$

Step3: Find Corner Frequency

Factor	CF
10	NIL
$(j\omega)^{-1}$	NIL
$(1 + 0.1j\omega)$	$\omega = \frac{1}{T} = \frac{1}{0.1} = 10 \text{ rad/s}$
$(1 + 0.5j\omega)^{-1}$	$\omega = \frac{1}{T} = \frac{1}{0.5} = 2 \text{ rad/s}$
$(1 + 0.2j\omega)^{-1}$	$\omega = \frac{1}{T} = \frac{1}{0.2} = 5 \text{ rad/s}$

Step 4 : Arrange CF in increasing order

Factor	CF (rad/s)	Slope (dB/decade)	Net slope (dB/decade)	Magnitude (dB)	Phase angle (\emptyset)
10	NIL	0	0	$20 \log_{10}(10)$	0^0
$(j\omega)^{-1}$	NIL	-20	-20	$-20 \log_{10}(\omega)$	-90^0
$(1 + 0.5j\omega)^{-1}$	2	-20	-40	$-20 \log_{10} \sqrt{1 + (0.5\omega)^2}$	$-\tan^{-1}\left(\frac{0.5\omega}{1}\right)$
$(1 + 0.2j\omega)^{-1}$	5	-20	-60	$-20 \log_{10} \sqrt{1 + (0.2\omega)^2}$	$-\tan^{-1}\left(\frac{0.2\omega}{1}\right)$
$(1 + 0.1j\omega)$	10	20	-40	$20 \log_{10} \sqrt{1 + (0.1\omega)^2}$	$+\tan^{-1}\left(\frac{0.1\omega}{1}\right)$

Step 5 : Find magnitude

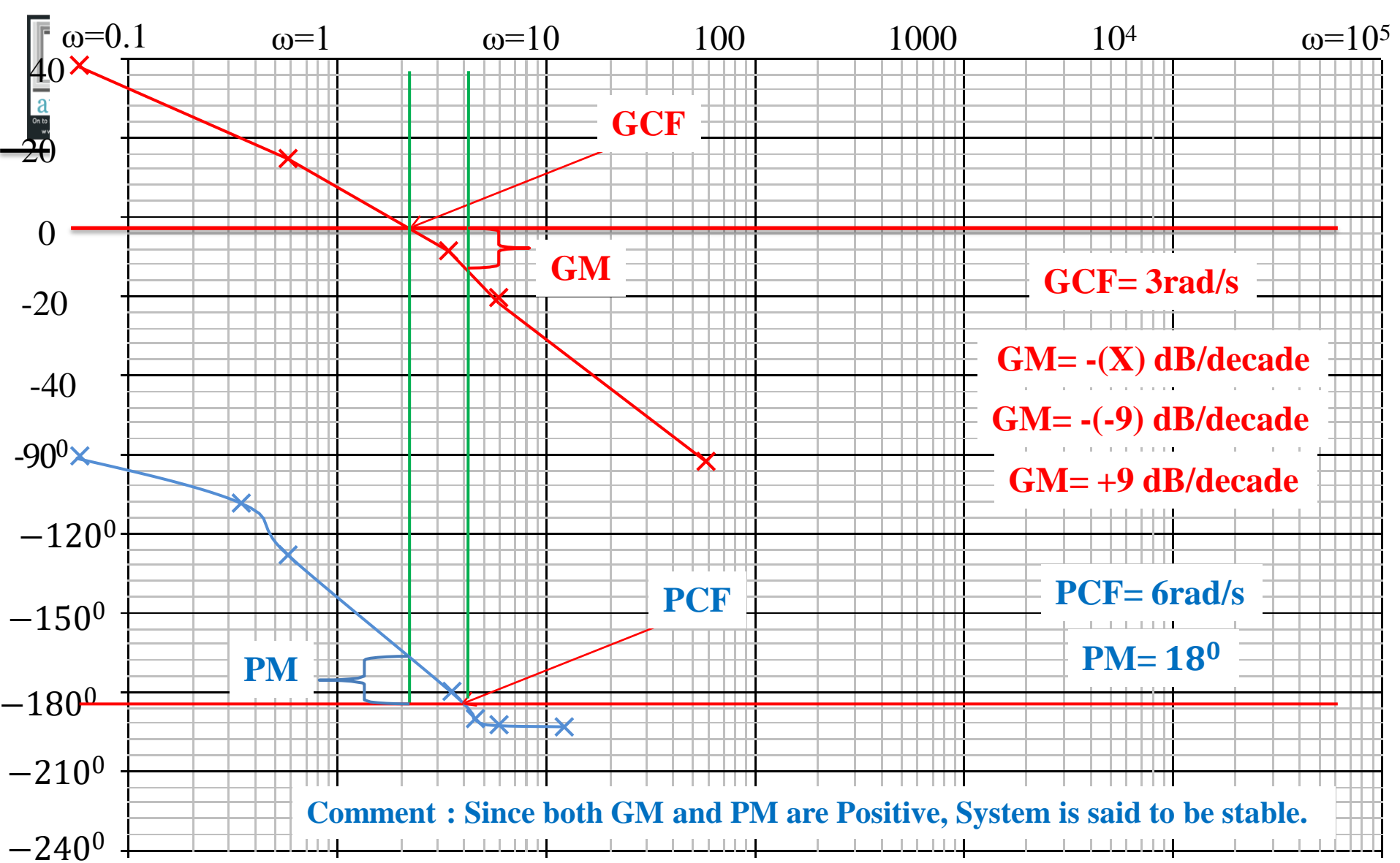
$$M = 20 \log_{10}(10) - 20 \log_{10}(\omega) - 20 \log_{10} \sqrt{1 + (0.5\omega)^2} - 20 \log_{10} \sqrt{1 + (0.2\omega)^2} + 20 \log_{10} \sqrt{1 + (0.1\omega)^2}$$

ω	Magnitude M (dB)
0.1	40
1	17.56
5	-4.62
10	-18.13
100	-59.96

Step 6 : Find phase angle

$$\phi = 0^\circ - 90^\circ - \tan^{-1} \left(\frac{0.5\omega}{1} \right) - \tan^{-1} \left(\frac{0.2\omega}{1} \right) + \tan^{-1} \left(\frac{0.1\omega}{1} \right)$$

ω	Phase Angle ϕ (deg)
0.1	-93.43°
0.5	-107°
1	-122.16°
5	-176.63°
6	-180.79°
8	-185.29°
10	-187.66°
20	-186.81°



Nyquist stability criteria :

Nyquist plot are the continuation of polar plot for finding stability of a closed loop control system by varying the values of ' ω ' from " $-\infty$ " to " ∞ "

Nyquist stability criteria is based on Cauchy's theorem of complex variable, popularly known as “*Principle of argument*”.

The importance of Nyquist stability lies in the fact that it can also be used to determine the relative degree of system stability by producing the so-called phase and gain stability margins.

Nyquist stability criteria is based on Cauchy's theorem

For a SISO feedback system the closed-loop transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \Rightarrow \quad F(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$D(s) = 1 + G(s)H(s)$$

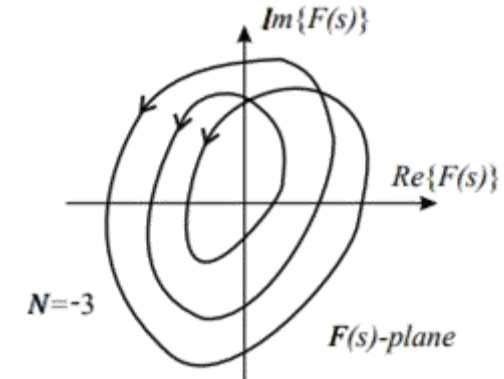
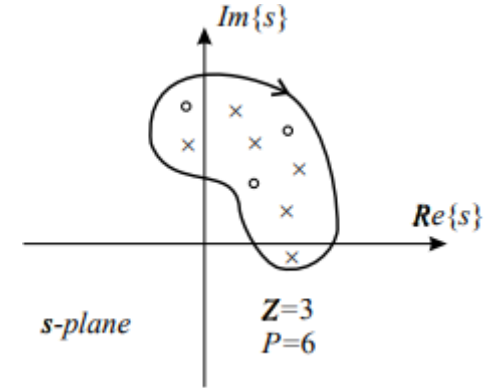
The Nyquist stability test is obtained by applying the Cauchy principle of argument to the complex function **$D(s)$** .

Cauchy's Principle of Argument

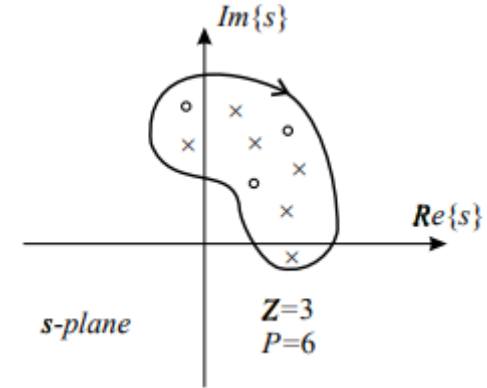
Let „ $F(s)$ ’ be an analytic function in a closed region of the complex plane given in Figure, except at a finite number of points.

Number of encirclements, $N = P - Z$

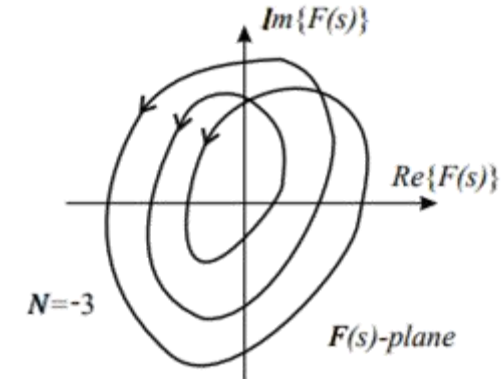
Then, as ‘ s ’ travels around the contour in the *s -plane* in the clockwise direction, the function $F(s)$ encircles the origin in the $(\text{Re}\{F(s)\}, \text{Im}\{F(s)\})$ – Plane in the same direction N times



If the enclosed „*S-plane*“ closed path contains only poles, then the direction the encirclement in the $G(s)H(s)$ plane will be opposite to the direction of the enclosed path in the “*S-plane*”.



If the enclosed „*S-plane*“ closed path contains only zero's, then the direction of encirclement in the $G(s)H(s)$ plane will be in the same direction as that of enclosed closed path in the “*S-plane*”.



Previous year VTU Question paper

[Dec-Jan 2019](#)

[Model OP1](#)

[Model OP2](#)

Any Questions ...?



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thank you